Tests of Hypothesis: One Sample

8.1 Determining the Claim, Null and Alternative Hypotheses

To complete this section of homework watch Chapter Eight, Lecture Examples: 117 and 118.

For problems 1 – 6, examine the given statement and express the null and alternative hypothesis in symbolic form:

1. The average weight loss obtained on the Atkins Diet is greater than 4 pounds. VS
2. The average grade in this class will be at least a 74. VS
3. The average waist circumference of adult males is 36.5 inches. VS
4. The average American has less than $10,000 dollars of savings. VS
5. The average length of time to eliminate a cold from the body is at most 14 days. VS
6. The average age of college graduates on the day of their graduation is not 21. VS

8.1 Answers

1. $H_0 : \mu \leq 4$
   $H_A : \mu > 4$
2. $H_0 : \mu \geq 74$
   $H_A : \mu < 74$
3. $H_0 : \mu = 36.5$
   $H_A : \mu \neq 36.5$
4. $H_0 : \mu \geq \$10,000$
   $H_A : \mu < \$10,000$
5. $H_0 : \mu \leq 14$
   $H_A : \mu > 14$
6. $H_0 : \mu = 21$
   $H_A : \mu \neq 21$

Need more exercises?

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

8.2 Critical Values for the Rejection Region

To complete this section of homework watch Chapter Eight, Lecture Example 120.

In the following eight exercises, use the given info from a hypothesis test to find the critical z-values:

7. Claim: $\mu = 76$, $\alpha = 0.01$ VS
8. Claim: $\mu < 1.234$, $\alpha = 0.05$ VS
9. Claim: $\mu > 100$, $\alpha = 0.02$ VS
10. Claim: $\mu \neq 68$, $\alpha = 0.02$ VS
11. Claim: $\mu \geq 24$, $\alpha = 0.01$ VS
12. Claim: $\mu \leq 73$, $\alpha = 0.10$ VS
13. Claim: $\mu < 890$, $\alpha = 0.005$ VS
14. Claim: $\mu < 14.8$, $\alpha = 0.04$ VS

8.2 Answers

7. -2.576, 2.576
8. -1.645
9. 2.05
10. -2.326, 2.326
11. -2.326
12. 1.282
13. -2.576
14. -1.75

Need more exercises?

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

8.3 Large-Sample Test of Hypothesis about a Population Mean

To complete this section of homework watch Chapter Eight, Lecture Examples 122 and 123 and Concept 7 in section 8.3.

In the next three problems, state the final conclusion for the hypothesis test:

15. Original claim: The average grade in the class is a 76. Initial Conclusion: Reject the null.

16. Original claim: The average IQ is greater than 100. Initial Conclusion: Do not reject the null.

17. Original claim: The average time to finish exam one is less than 75 minutes. Initial Conclusion: Reject the null.

18. A consumer group claims that the Doritos snack pack size has an average weight below 1.75 ounces which is the weight labeled on the bags. A random sample of 49 bags had an average weight of 1.71 ounces and a standard deviation of 0.13 ounces. At the 5% significance level, test the consumer group's claim. Give the practical interpretation of the outcome of the test. VS

19. A professor claims that it takes the average student no more than 40 minutes to finish his final exam. A random selection of 39 students was timed while taking the final. The students had an average completion time of 41.6 minutes and a standard deviation of 6 minutes. Use a 1% significance level to test the professor's claim. Give the practical interpretation of the outcome of the test. VS

20. A gym owner claims that the average adult male has a waist measurement equal to 36 inches. A study of 42 males showed the average waist size to be 35.9 inches and a standard deviation of 3.33 inches. At the 2% significance level, test the gym owner's claim. Give the practical interpretation of the outcome of the test. VS

21. The US government claims that the average woman has a mean weight of 143 pounds. A study is done which involved a random sample of 35 women with an average weight of 146 pounds and a standard deviation of 29 pounds. Use a 1% significance level to test the government's claim. Give the practical interpretation of the outcome of the test. VS

22. A physician claims that the average male weighs less than 180 pounds. A sample of 32 randomly selected males has an average weight of 172 pounds and a standard deviation of 29 pounds. Use a 5% significance level to test the physician's claim. Give the practical interpretation of the outcome of the test.

23. A sociologist claims that marriages that end in divorce on average last 6 years. A study of 35 divorced couples revealed an average length of their marriages to be 8.2 years with a standard deviation of 1.33 years. As a part of that study, the researchers constructed a 98% confidence interval for the true mean length of marriage in years for divorced couples. That interval was from 7.7 years to 8.7 years. Use the confidence interval to test the sociologist's claim at the 2% significance level. Give the practical interpretation of the outcome of the test.

8.3 Answers

15. There is sufficient evidence to reject the claim that the average grade in the class is a 76.

16. There is not sufficient evidence to support the claim that the average IQ is greater than 100.

17. There is sufficient evidence to support the claim that the average time to finish exam one is less than 75 minutes.

18. $Claim: \mu < 1.75$, $\begin{matrix} H_0: \mu \geq 1.75 \\ H_A: \mu < 1.75 \end{matrix}$, $TestStat: Z = -2.15$, $CriticalValue(s): -1.645$,

$InitialConclusion$: Reject the null, support the alternative

There is sufficient evidence to support the claim…

Practical outcome: Doritos is under filling its bags.

19. $Claim: \mu \leq 40$, $\begin{matrix} H_0: \mu \leq 40 \\ H_A: \mu > 40 \end{matrix}$, $TestStat: Z = 1.67$, $CriticalValue(s): 2.326$,

$InitialConclusion$: Do not reject the null, do not support the alternative

There is not sufficient evidence to reject the claim…

Practical outcome: The professor is correct.

20. $Claim: \mu = 36$, $\begin{matrix} H_0: \mu = 36 \\ H_A: \mu \neq 36 \end{matrix}$, $TestStat: Z = -0.19$, $CriticalValue(s): -2.326, 2.326$,

$InitialConclusion$: Do not reject the null, do not support the alternative

There is not sufficient evidence to reject the claim…

Practical outcome: The males have an average waist measurement of 36 inches.

21. $Claim: \mu = 143$, $\begin{matrix} H_0: \mu = 143 \\ H_A: \mu \neq 143 \end{matrix}$, $TestStat: Z = 0.61$, $CriticalValue(s): -2.576, 2.576$,

$InitialConclusion$: Do not reject the null, do not support the alternative

There is not sufficient evidence to reject the claim…

Practical outcome: The females have an average weight of 143 pounds.

22. $Claim: \mu < 180$, $\begin{matrix} H_0: \mu \geq 180 \\ H_A: \mu < 180 \end{matrix}$, $TestStat: Z = -1.56$, $CriticalValue(s): -1.645$,

$InitialConclusion:$ Do not reject the null, Do not support the alternative

There is not sufficient evidence to support the claim…

Practical outcome: This evidence can't be used to argue men on average weigh less than 180 pounds.

23. Since 6 years is not a part of the given interval which says the average is from 7.7 to 8.7 years, we should reject the sociologist's claim, so it seems these failed marriages last longer than 6 years.

Need more exercises?

8.4 Observed Significance Levels: p-Values

To complete this section of homework watch Chapter Eight, Lecture Examples 124, 125, and 126.

In the following seven problems, use the given information to find the p-value:

24. Claim: $\mu < 36$ Test Stat: $z = -2.13$ VS
25. Claim: $\mu > 84$ Test Stat: $z = 1.89$ VS
26. Claim: $\mu \leq 110$ Test Stat: $z = 2.05$
27. Claim: $\mu \geq 55$ Test Stat: $z = -1.10$
28. Claim: $\mu = 1.287$ Test Stat: $z = 2.89$ VS
29. Claim: $\mu \neq 36$ Test Stat: $z = -2.56$
30. Claim: $\mu < 15$ Test Stat: $z = 1.58$ VS

31. The amount of time to finish the US census is of interest to the federal government. A member of the Census bureau claims it takes no more than ten minutes to fill out the census. A sample of 52 randomly chosen citizens were timed while completing the census. They had a mean of 10.6 minutes and a standard deviation of 2.25 minutes. Use a 5% significance level and the p-value method to test the claim from the member of the census bureau.

32. Some researchers think that divorce is more likely when a couple marries at a young age. One researcher claims that the average divorced male was younger than 25 on the day of his wedding. A study of 33 divorced males shows their average age on the day of their wedding was 24.3 with a standard deviation of 2.5 years. At the 3% significance level, use the p-value method to test the researcher's claim. VS

33. A speed reading teacher claims that it takes the average reader more than ten hours to finish a 300 page book. Thirty-one randomly selected readers were given 300 page novels to read and they timed themselves. The average completion time for the group was 11 hours with a standard deviation of 2.11 hours. At the 4% significance level, use the p-value method to test the researcher's claim.

34. Historically, the average height for males was believed to be 68 inches. A doctor believes the average height has increased over the last 100 years. He claims the average male is now 70 inches tall. A random sample of 50 men had an average height of 68.9 inches and a standard deviation of 2.8 inches. Use a 10% significance level and the p-value method to test the doctor's claim. VS

8.4 Answers

24. P = 0.0166

25. P = 0.0294

26. P = 0.0202

27. P = 0.1357

28. P = 0.0038

29. P = 0.0104

30. P = 0.9429

31. $Claim: \mu \le 10$, $\begin{matrix} H_0: \mu \le 10 \\ H_A: \mu > 10 \end{matrix}$, $TestStat: Z = 1.92$, $PValue: 0.0274$,

$InitialConclusion$: Reject the null, Support the alternative

There is sufficient evidence to reject the claim…

32. $Claim: \mu < 25$, $\begin{matrix} H_0: \mu \ge 25 \\ H_A: \mu < 25 \end{matrix}$, $TestStat: Z = -1.61$, $PValue: 0.0537$,

$InitialConclusion$: Do not reject the null, do not support the alternative

There is not sufficient evidence to support the claim…

33. $Claim: \mu > 10$, $\begin{matrix} H_0: \mu \le 10 \\ H_A: \mu > 10 \end{matrix}$, $TestStat: Z = 2.64$, $PValue: 0.0041$,

$InitialConclusion$: Reject the null, support the alternative

There is sufficient evidence to support the claim…

34. $Claim: \mu = 70$, $\begin{matrix} H_0: \mu = 70 \\ H_A: \mu \neq 70 \end{matrix}$, $TestStat: Z = -2.78$, $PValue: 0.0054$,

$InitialConclusion$: Reject the null, support the alternative

There is sufficient evidence to reject the claim…

Need more exercises?

8.5 Small-Sample Test of Hypothesis about a Population Mean

To complete this section of homework watch Chapter Eight, Lecture Examples 127 and 128.

35. Degrees of freedom: When using a t table to find critical values, we must use the appropriate number of degrees of freedom. If a sample consists of ten values, what is the degrees of freedom for this problem? If you didn't know any of the ten values, but you knew their mean was 100, how many values could you make up before the remaining values are determined by the restriction that the mean is 100?

36. An accountant claims that the average hourly wage for pizza delivery drivers is more than $10. If a sample of 28 pizza delivery driver's paychecks has a mean of $9.25 and a standard deviation of $1.00, why is it not necessary to conduct a formal hypothesis test of the accountant's claim?

37. The CEO of Equifax credit reporting agency claims the average credit rating has dropped below 675 points. A study of 20 randomly selected credit scores had an average of 660 points and a standard deviation of 95.3 points. Use a 5% significance level to test the claim that credit scores are now on average below 675 points. The CEO claims the results are not valid since they came from too small a sample. Is there any merit to his argument? VS

38. The Natural Foods Diet claims that people lose an average of ten pounds in two months on the plan. A random sample of 26 people lost an average 8.9 pounds on the diet in two months. The standard deviation was 3.25 pounds. Use a 2% significance level to test the claim that the diet helps people lose an average of 10 pounds in two months. VS

39. A female student of mine claims that the average height of female super models is the same as the average for women in general (64 inches). I randomly selected 9 supermodels from a list of super models and found they had an average height of 70.2 inches with a standard deviation of 2.5 inches. Use a 1% significance level to test my student's claim. If she disputes the result of the test by arguing that I took too small a sample, is there some merit to her argument?

40. The lifespan for the general population of males born in 1980 is 77 years old. A worker for the Census Bureau claims that the average lifespan for college professors is greater than 77. A random sample of 17 deceased college professors had a mean lifespan of 89 and a standard deviation of 9.5 years. Use a significance level of 10% to test the CB worker's claim. VS

41. The average undergrad cost for tuition, fees, room and board at two-year colleges 5 years ago was $13,252. This year a random sample of 20 schools had a mean (adjusted for inflation) of $15,560 and a standard deviation of $3,500. Use a 1% significance level to test the claim that tuition, fees, and room and board (adjusted for inflation) at two-year colleges has risen over the past 5 years. VS

8.5 Answers

35. 9; 9 because to have an average of 100 you'd need a sum of 1000 (1000/10 = 100), so you can pick any random 9 numbers. Say I chose: 1, 2, 3, 4, 5, 6, 7, 8, 9, the sum of these is 45. Thus to have a sum of 1000 my last number would have to be 955 (I wasn't free to choose this last value. It had to be 955 or I wouldn't have an average of 100 for the ten numbers.).

36. He is trying to support his claim that it is over $10, but his evidence is lower than $10 dollars, how can this ever provide support for his argument?

37. $Claim: \mu < 675$, $\begin{matrix} H_0: \mu \geq 675 \\ H_a: \mu < 675 \end{matrix}$, $TestStat: -0.70$, $CriticalValue: -1.729$,

$InitialConclusion$: Do not reject the null, do not support the alternative

$FinalConclusion$: There is not sufficient evidence to support the claim…

The t-test is a less powerful test than the z-test, so when the t-test does not reject the null it is possible that it is because it is not powerful enough. However, if we can reject the null with the t-test then we do not need to worry that the test was too weak. In this case, it is possible the test was too weak to detect the shift lower in credit scores, but the test stat wasn't very extreme at all. Probably, the z-test would have the same conclusion.

38. $Claim: \mu = 10$, $\begin{matrix} H_0: \mu = 10 \\ H_a: \mu \neq 10 \end{matrix}$, $TestStat: -1.73$, $CriticalValues: -2.485, 2.485$,

$InitialConclusion$: Do not reject the null, do not support the alternative

$FinalConclusion$: There is not sufficient evidence to reject the claim…

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

39. $Claim: \mu = 64$, $\begin{matrix} H_0: \mu = 64 \\ H_a: \mu \neq 64 \end{matrix}$, $TestStat: 7.44$, $CriticalValues: -3.355, 3.355$,

$InitialConclusion$: Reject the null, support the alternative

$FinalConclusion$: There is sufficient evidence to reject the claim…

Remember, the t-test is a less powerful test than the z-test, so when the t-test rejects the null it means the z-test would almost certainly also reject the null. If we can reject the null with the t-test then we do not need to worry that the test was too weak. The student's complaint has no merit.

40. $Claim: \mu > 77$, $\begin{matrix} H_0: \mu \leq 77 \\ H_a: \mu > 77 \end{matrix}$, $TestStat: 5.21$, $CriticalValue: 1.337$,

$InitialConclusion$: Reject the null, support the alternative

$FinalConclusion$: There is sufficient evidence to support the claim…

41. $Claim: \mu > \$13,252$, $\begin{matrix} H_0: \mu \leq \$13,252 \\ H_a: \mu > \$13,252 \end{matrix}$, $TestStat: 2.95$, $CriticalValue: 2.539$,

$InitialConclusion$: Reject the null, support the alternative

$FinalConclusion$: There is sufficient evidence to support the claim…

8.6 Hypothesis about a Population Proportion

To complete this section of homework watch Chapter Eight, Lecture Examples 129 and 130.

42. An educator in Michigan estimates the dropout rate for seniors in high school to be 15%. Last year, 38 seniors out of 201 seniors dropped out. Use the p-value method and a 5% significance level to determine if we can reject the educator's claim. VS

43. The government mint claims that at least 77% of the public is against changing dollar coins for dollar bills. In a survey of 800 people, 550 said they were opposed to the change. At the 5% level of significance, test the mint's claim.

44. Nationally 60% of Ph.D. students have paid assistantships. An FIU professor thinks at FIU the rate is lower than this. In a random sample of 50 Ph.D. students, 26 have assistantships. Using a 5% significance level, test the FIU professor's claim. VS

45. A senatorial candidate claims that most of the people in the country feel they are worse off today than they were two years ago. A poll of 500 people in the country reveals that 255 feel they are worse off today than two years ago. At the 10% significance level, test the senatorial candidate's claim. VS

46. A professor claims that at most 10% of the class gets A's each semester in his course. A random sample of 100 students from previous terms show that he gave out 15 A's. Using a 5% significance level, test the professor's claim.

8.6 Answers

42. $Claim: \rho = 15\%$, $\begin{matrix} H_0: \rho = 15\% \\ H_a: \rho \neq 15\% \end{matrix}$, $TestStat: 1.55$, $PValue: 0.1212$,

$InitialConclusion$: Do not reject the null, do not support the alternative

$FinalConclusion$: There is not sufficient evidence to reject the claim…

43. $Claim: \rho \geq 77\%$, $\begin{matrix} H_0: \rho \geq 77\% \\ H_a: \rho < 77\% \end{matrix}$, $TestStat: -5.54$, $CriticalValue: -1.645$,

$InitialConclusion$: Reject the null, support the alternative

$FinalConclusion$: There is sufficient evidence to reject the claim…

44. $Claim: \rho < 60\%$, $\begin{matrix} H_0: \rho \geq 60\% \\ H_a: \rho < 60\% \end{matrix}$, $TestStat: -1.15$, $CriticalValue: -1.645$,

$InitialConclusion$: Do not reject the null, do not support the alternative

$FinalConclusion$: There is not sufficient evidence to support the claim…

45. $Claim: \rho > 50\%$, $\begin{matrix} H_0: \rho \leq 50\% \\ H_a: \rho > 50\% \end{matrix}$, $TestStat: 0.45$, $CriticalValue: 1.282$,

$InitialConclusion$: Do not reject the null, do not support the alternative

$FinalConclusion$: There is not sufficient evidence to support the claim…

46. $Claim: \rho \leq 10\%$, $\begin{matrix} H_0: \rho \leq 10\% \\ H_a: \rho > 10\% \end{matrix}$, $TestStat: 1.67$, $CriticalValue: 1.645$,

$InitialConclusion$: Reject the null, support the alternative

$FinalConclusion$: There is sufficient evidence to reject the claim…

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

8.7 Type I and Type II Error Probabilities

To complete this section of homework watch Chapter Eight, Lecture Example 130.5.

47. If you have a significance level of 1% and a p-value of 0.0238, after forming the appropriate conclusion, what possible error might you have committed (Type one or two)? Explain. VS

48. If you are testing the claim: $\mu > 96$, and your significance level is 8%, what is the probability that you commit the type one error? VS

49. If you are testing the claim: $\mu = 200$, and your significance level is 5%, what is the probability that you commit the type one error? VS

8.7 Answers

47. Since the p-value is greater than alpha, we do not reject the null. If we do not reject the null, we may have just committed a type two error.

48. At most 8% (one-tail test)

49. Exactly 5% (two-tail test)

Take a sample exam for chapter 7 & 8

Chapter 8 Mixed Review

50. A bank manager wants to reduce the average wait time at her branch. She institutes a new system for her workers to follow. The previous average wait time was five minutes. The manager randomly samples 10 customer wait times at the bank while the new system is in place. She uses the results to perform a hypothesis test of the claim that the new mean is lower than five minutes. The results are as follows:

$H_0 : \mu \geq 5$ $H_A : \mu < 5$ **Test statistic: t = -4.302. Critical value: t = -2.262. Reject H_0. There is sufficient evidence to support the claim that the new mean is lower than five minutes.**

The assistant manager suggests that the results may not be valid since the sample size was very small. Is the assistant manager correct?

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

51. A doctor claims that the average woman is not more than 63 inches tall. A biologist disagrees. A sample of 47 women has a mean of 64.1 inches and a standard deviation of 2.3 inches. State the null and alternative hypotheses necessary to test the doctor's claim. If we were to decide to reject the null hypothesis, how would we state the final conclusion?

52. A candy store decides to test if a new layout to the gummy candy bins will increase the amount of gummy candy bought by the average customer. The previous mean weight of customer bags was 8.3 ounces. The manager wants to conduct a hypothesis test at the 5% significance level to determine if the new mean weight is different from 8.3 ounces. If the manager decides to conduct this test at the 1% significance level instead and she changes nothing else about the hypothesis test, what effect will lowering the significance level have on the likelihood of committing an error?

53. A man claims that if he approaches different women at the supermarket using the same pickup line, he will obtain the telephone numbers of 25% of the women. After testing this tactic on 200 randomly chosen women at the supermarket, he has obtained 18 phone numbers. Use this information to calculate a test statistic that would be used to test the man's claim.

54. A professor claims that it takes the average student no more than 40 minutes to finish his final exam. A random sample of 39 students was timed while taking the final. The students had an average completion time of 41.6 minutes and a standard deviation of 6 minutes. If we test the professor's claim, what would the p-value be for this test?

55. A mechanic claims that the average lifetime for power window motors is less than 7 years. State the null and alternative hypotheses for a test of the mechanic's claim at the 2.5% significance level. What is the type I error for this test, and what is the probability that the type I error occurs?

56. A physician claims that the average male weighs 178 pounds. A sample of 50 randomly selected males has an average weight of 172 pounds and a standard deviation of 29 pounds. Use a 5% significance level and the p-value to test the physician's claim. Based on your conclusion, what is the probability that we committed a type I error?

57. An accountant claims that the average hourly wage for waiters is more than $18. If a random sample of 24 waiters' wages has a mean of $19.25 and a standard deviation of $1.75, what is the critical value and rejection region for a test of the accountant's claim at the 5% significance level?

58. A surgeon claims that the average patient undergoing breast augmentation surgery is 32 years old. A study of 26 randomly selected patient records is conducted. The mean age for the sample is 30.5 years, and the standard deviation is 4.9 years. Use a 2% significance level to test the surgeon's claim.

Chapter 8 Mixed Review Answers

50. No, the assistant manager is not correct. If the sample data allows us to reject the null hypothesis, the sample size was sufficient.
51. $H_0 : \mu \leq 63, H_A : \mu > 63$; Final conclusion: The sample data allow us to reject the claim that the average woman is no more than 63 inches tall.
52. Lowering the significance level will reduce the likelihood of committing the type I error to a 1% chance, but it will also increase the likelihood of committing the type II error.
53. Test statistic: Z = $\frac{.09 - 0.25}{\sqrt{\frac{.25 * .75}{200}}}$ = -5.23
54. Z = 1.67; p = .0475 (the p-value is equal to the area to the right of 1.67, which is 0.0475)
55. $H_0 : \mu \geq 7, H_A : \mu < 7$; The type I error would be the error of rejecting the hypothesis that the mean is greater than or equal to 7 years when it actually is greater than or equal to 7 years. This error has **at most** a 2.5% chance of occurring.
56. $H_0 : \mu = 178, H_A : \mu \neq 178$; Test stat: z = -1.46; p-value: 14.42% (twice the area to the left of -1.46); Do not reject the null hypothesis. Since we did not reject the null hypothesis, there is zero chance we committed the type one error.
57. Critical value: t = 1.714; Rejection region: reject if the test statistic is greater than 1.714.
58. $H_0 : \mu = 32, H_A : \mu \neq 32$; Test stat: t = -1.56; Critical t-values: $\pm$ 2.485; Do not reject the null hypothesis; There is not sufficient evidence to reject the surgeon's claim.