Confidence Intervals

7.1 Finding Critical Z Values

To complete this section of homework watch Chapter Seven, Lecture Examples: 100 and 101-103.

Directions: Find the critical $Z_{\alpha/2}$ value given the following confidence levels.

1. 98% VS
2. 99% VS
3. 95% VS
4. 90% VS
5. 92% VS
6. 96% VS
7. 94% VS
8. 97% VS

7.1 Answers:

1. 2.326
2. 2.576
3. 1.960
4. 1.645
5. 1.75
6. 2.05
7. 1.88
8. 2.17

Need more exercises?

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

7.2   Large-Sample Confidence Intervals for a Population Mean

To complete this section of homework watch Chapter Seven, Lecture Examples: [104](#) and [105](#).

9. A 99% confidence interval was constructed to estimate the mean, and the resulting interval is [9.68, 11.02]. Write a statement to correctly interpret the interval.

10. Interpret the confidence coefficient used in problem 9. In other words, what does it mean to say we are 99% confident?

11. If an estimator is unbiased, what does that mean in plain terms?

For the following four problems, calculate the Margin of Error that would be used when estimating the mean for the following scenarios:

12. The confidence level is 99%. The sample size is 37. The sample mean is 25, and the population standard deviation is 4.

13. The confidence level is 92%. The sample size is 49. The sample mean is 512, and the population standard deviation is 28.

14. The confidence level is 90%. The sample size is 100. The sample mean is 69, and the population standard deviation is 3.2.

15. The confidence level is 94%. The sample size is 32. The sample mean is 100, and the population standard deviation is 14.6.

For the following four problems, use the provided information to construct a confidence interval to estimate the population mean.

16. Salaries of college statistics professors in 2015 at Florida public universities: Confidence level = 95%, n =36, $\bar{x}$ =\$85,113, and σ =\$11,024. [VS](#)

17. Speeds of drivers ticketed in a 65mph zone: Confidence level = 98%, n =31, $\bar{x}$ =81, and σ =3.4. [VS](#)

18. Credit scores: Confidence level = 91%, n =48, $\bar{x}$ =688, and σ =67.

19. Losses of patrons at the Seminole Casinos: Confidence level = 93%, n =50, $\bar{x}$ =\$170, and σ = \$89.

20. A survey of thirty-one 2005 Major League Baseball salaries for pitchers playing in the National League had a mean of \$2,522,785 and a standard deviation of \$4,065,579. Construct the 98% confidence interval for the true average salary for all NLMLB pitchers in 2005. [VS](#)

: indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com

21. What does it mean that the standard deviation is so large for the sample data in the previous problem?

22. A transportation agency in Miami is interested in estimating the average commute time for residents of the city. They collected a random sample of 500 commuters and recorded the following summary statistics for their daily commute times (in minutes):

    Sample Mean ($\bar{x}$): 28.6 minutes

    Sample Standard Deviation (s): 6.2 minutes

    Assuming a normal distribution, calculate a 90% confidence interval for the true average commute time for residents of Miami. The mayor claims commute times are now less than 30 minutes on average. Is this claim supported by the data in this problem?

23. If the standard deviation is large for a set of data that is used to create a confidence interval, what effect will this have on the margin of error? How can this effect be countered?

24. The following interval is an interval estimate of the true mean score obtained by women taking the first exam in a STA 3033 class: (72.1, 77.4). A similar interval was created for men taking STA 3033. The result was as follows: (68.9, 73.1). Compare the two intervals. Can we say that the average woman taking STA 3033 outscores the average man taking STA 3033? Explain.

25. An environmental agency is interested in estimating the average fuel economy of brand-new Honda Accord hybrid vehicles produced in 2017. They collected a random sample of 49 vehicles and recorded the summary statistics of their measured miles per gallon (MPG). The sample mean for the 49 cars was 48.5 MPG. The sample standard deviation was 2.2 MPG. Calculate a 95% confidence interval for the true average fuel economy of brand-new Honda Accord hybrid vehicles in 2017. Honda claims the average fuel economy for the 2017 Accord hybrid is 48 MPG. Does the interval contradict that claim?

26. The following interval is an interval estimate of the true mean score obtained by Biology majors taking the first exam in a STA 3111 class: (78.2, 84.4). A similar interval was created for Psychology majors taking STA 3111. The result was as follows: (67.9, 76.8). Compare the two intervals. Can we say that the average Biology major taking STA 3111 outscores the average Psychology major taking STA 3111? Explain.

27. A researcher is interested in estimating the average attention span of students during a lecture. The researcher collected a random sample of 64 students and recorded the summary statistics of their attention span times (in minutes). The students had a sample mean attention span of 19.8 minutes and a standard deviation of 2.2 minutes. Assuming a normal distribution, calculate a 98% confidence interval for the true average attention span of students during a lecture.

28. A 1993 study of in-state annual tuition rates at public schools in the US drew a random sample of 56 public colleges and universities that had a mean of $2,319 and a standard deviation of $1,136. Construct the 95% confidence interval for the true mean annual, in-state tuition costs for US colleges and universities in 1993. In an interview from 1993, a college president claimed the $3,527 per year in-state tuition that his college charged was average for public schools. Does the interval you created contradict his claim? 📷 VS

29. A study of children's weights was conducted in Hong Kong. A sample of forty children each 68 inches tall had a mean weight of 128.9 pounds and a standard deviation of 9.2 pounds. Create a 96% confidence interval for the true mean weight of children that are 68 inches tall in Hong Kong. A Chinese researcher claims that the average child in Hong Kong is underweight. If the cut off weight for being underweight at 68 inches tall is 122 pounds, does this interval provide any evidence to contradict the claim that children in Hong Kong are underweight? 📷 VS

30. The following Minitab display was created from speed data collected on a strip of I-95 where the speed limit is posted as 55mph. Use the Minitab display below to identify the point estimate for the population mean and interpret the 95% confidence interval provided by Minitab.

MINITAB

Variable	N	Mean	StDev	SE Mean	95% CI
Speed	81	67.3849	3.3498	0.3722	(66.6554, 68.1144)

7.2   Answers:

9. There is 99% confidence that the true population mean μ is inside the interval from 9.68 to 11.02.

10. It means with repeated sampling, 99% of similarly created intervals would capture the population mean.

11. Values of an unbiased estimator cluster around the target parameter, which means they do not systematically overestimate or underestimate. The average value of the unbiased estimator is the target parameter.

12. 1.69

13. 7

14. 0.5264

15. 4.852

16. [$81,511.83, $88,714.17]

17. [79.58, 82.42]

18. [671.6, 704.4]

19. [147.2, 192.8]

20. [$824,340.86, $4,221,229.14] We are 98% confident that the true mean salary for NLMLB pitchers is between 0.8 million and 4.2 million dollars per year.

21. It means there is a lot of variation in the salaries of NLMLB pitchers. Several players are paid very large amounts of money, while others are paid relatively small sums.

22. [28.6 – 1.645*6.2/root(500), 28.6 + 1.645*6.2/root(500)] = [28.14, 29.06] We are 90% confident that the true mean commute time in Miami is between 28.14 minutes and 29.06 minutes. Since the upper bound of the interval is less than 30 minutes, this result supports the mayor's claim.

23. The large standard deviation will make the margin of error large, which means the confidence interval will be wide. To counter this, a researcher should increase the sample size.

24. Based on the intervals, it is inconclusive. We cannot say women perform better on average. The lowest value in the confidence interval for the women is inside the confidence interval for the men. This indicates there are values that both intervals share (for example 72.9). Any shared values in the intervals could be the mean for both groups. Also, it is equally plausible to say that the mean for women is higher and that the mean for men is higher and to say that the two means are the same.

25. [48.5 – 1.960*2.2/root(49), 48.5 – 1.960*2.2/root(49)] = [47.88, 49.12] We are 95% confident that the true mean MPG for 2017 Accord hybrids is between 47.88 mpg and 49.12 mpg. The interval does not contradict Honda's claim since 48 MPG is inside of the interval. This indicates 48 MPG could be the mean.

26. Yes, you can conclude that Biology majors outscore Psychology majors. The two intervals do not overlap at all, which means the lowest likely value for the mean for Biology majors is higher than the highest likely value for Psychology majors. However, it should be noted that if both intervals were at the 95% confidence level, the confidence in the resulting comparison is only 90.25% (0.95x0.95 = 0.9025).

27. [19.8 – 2.326*2.2/8, 19.8 + 2.326*2.2/8] = [19.16, 20.44] We are 98% confident that the true mean attention span for students is between 19.16 minutes and 20.44 minutes.

28. [$2,021, $2,617] This interval does contradict the claim of the college president since the candidate values for the average are all lower than the value he claimed.

29. [125.9, 131.9] Yes, the lowest number in the interval is higher than 122 lbs, so we do not have evidence that the children on average are underweight.

30. The point estimate for the population mean is the sample mean which is 67.3849. We are 95% confident that the true average speed of cars on this stretch of I-95 is between 66.7 and 68.1 miles per hour.

[Need more exercises?](#)

7.3   Determining the Sample Size

To complete this section of homework watch Chapter Seven, Lecture Examples: 106 and 106.5.

For the following four problems, use the information provided to determine the sample size needed to construct a confidence interval to estimate the population mean.

31. Margin of error: 0.19   Confidence level: 95%   Population standard deviation: 4   VS

32. Margin of error: 15   Confidence level: 90%   Population standard deviation: 102

33. Margin of error: 0.68   Confidence level: 98%   Population standard deviation: 3.9

34. Margin of error: 1   Confidence level: 99%   Population standard deviation: 2.8

35. A stockbroker on Wall Street wants to estimate the average daily-high price for JAVA Sun Microsystems stock. What sample size is necessary to form a 99% confidence interval to estimate the mean daily-high within 0.25 dollars? Assume the population standard deviation is known to be 4.944 dollars.   VS

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

36. A personal trainer wants to estimate the true average increase in chest circumference after a 12-week training program. If he wants to estimate the mean increase in circumference within a half inch using a 95% confidence interval, how many randomly chosen individuals will he need to measure? Assume the standard deviation is known to be 1.75 inches. ▣ VS

37. A college freshman wants to estimate the true time it takes to have his order completed at the Subway sandwich shop on campus. He has estimated that the standard deviation for order-completion times is 3.3 minutes. If he wants to estimate the true average, order-completion time to within one minute using a 95% confidence interval, how many times will he have to time himself purchasing a sandwich at Subway?

38. You want to estimate the mean weight loss of people one year after using the Atkins diet. How many dieters must be surveyed if we want to be 95% confident that the sample mean weight loss is within 0.25 lb of the true population mean? Assume that the population standard deviation is known to be 10.6 lb (based on data from a study published in JAMA, Vol. 293, No. 1).

7.3 Answers:

31. 1703

32. 126 (don't forget we always round sample size up)

33. 178

34. 53

35. 2,596

36. 48

37. 42

38. 6,907

Need more exercises?

7.4   Finding Critical T Values

To complete this section of homework watch Chapter Seven, Lecture Examples: 107 - 109.

Finding the appropriate critical value ($Z_{\alpha/2}$ or $t_{\alpha/2}$). For the following eight problems, find the appropriate critical value given the sample size and confidence level.

39. For n = 29 and a confidence level of 98% ▣ VS

▣ : *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

40. For n = 35 and a confidence level of 98% <u>VS</u>

41. For n = 19 and a confidence interval of 80% <u>VS</u>

42. For n = 128 and a confidence interval of 80%

43. For n = 15 and a confidence interval of 99% <u>VS</u>

44. For n = 100 and a confidence interval of 99%

45. For n = 27 and a confidence interval of 90% <u>VS</u>

46. For n = 49 and a confidence interval of 90%

47. You might have noticed the pattern in the eight questions above (t then Z, t then Z, …). You might also have noticed that for a given confidence level, the $t_{\alpha/2}$ is always larger than the $Z_{\alpha/2}$. What effect will this have on the corresponding confidence intervals?

7.4   Answers:

39. 2.467

40. 2.326

41. 1.330

42. 1.282

43. 2.977

44. 2.576

45. 1.706

46. 1.645

47. The t-intervals will always be wider than the z-intervals for the same given confidence level and sample size.

<u>Need more exercises?</u>

7.5	Small-Sample Confidence Intervals for a Population Mean

To complete this section of homework watch Chapter Seven, Lecture Examples: 110, 111, and 112.

48. Use the given values to find the margin of error and the confidence interval used to estimate the mean: Weight loss after a year of dieting: n = 29, $\bar{x} = 3.1$, $s = 4.25$, and C. level = 99% VS

49. Use the given values to find the margin of error and the confidence interval used to estimate the mean: Lifespan of a PC: n = 21, $\bar{x} = 6.2$, $s = 2.1$, and C. level = 95%

50. A sample of 28 healthy babies has an average birth weight of 6.83 lbs and a standard deviation of 1.53 lbs. Construct a 95% confidence interval for the true average weight for healthy babies. If another study revealed that babies exposed to cocaine prenatally had an average birth weight of 4.92 lbs, does it seem that cocaine might lead to lower than normal birth weights? VS

51. A medical study designed to determine how much water is needed to become properly hydrated was conducted using 26 subjects. The average amount of water required daily by the subjects to be fully hydrated was 53.6 ounces with a standard deviation of 10 ounces. Form a 90% confidence interval to estimate the true average amount of water needed to become fully hydrated. Does this interval contradict the long-standing belief that the average person requires 64 ounces of water per day to be fully hydrated? VS

52. For a study designed to measure the accuracy of stockbroker predictions, twenty-four brokers were asked to predict the net gain or loss of a particular stock over the course of one week. The brokers' predicted amount was then subtracted from the actual net gain or loss of the stock to get a prediction error. This yielded a mean error of 24 points and a standard deviation of 13 points. Construct a 98% confidence interval for the true average error made by stockbrokers. Does this interval contain zero? If it doesn't, what does that say about the average prediction error of the brokers?

53. I have kept track of my car's fuel consumption over the past month. I observed the mpgs for 17 trips from my home to FIU. The average mpgs for the 17 trips was 31.1 mpg with a standard deviation of 2.1 mpg. Form a 95% confidence interval for my true fuel consumption when driving to FIU. To keep my gas budget in check, I need to average at least 30 miles per gallon. Based on the interval you just created, should I slow down? VS

7.5	Answers:

48. [0.919, 5.281]

49. [5.244, 7.156]

50. [6.24, 7.42] Since 4.92 is outside of the interval it seems cocaine exposure lowers birth weight.

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

51. [50.3, 56.9] Since 64 ounces is outside this interval it seems that the average person does not need that much water.

52. [17.4, 30.6] It seems the brokers do not have an average error of zero since it is not in the interval. An average of zero would be good since that would mean no error on average, but this interval does not include zero. This means the brokers make prediction errors on average.

53. [30.02, 32.18] I don't need to drive slower because the worst case scenario is that my average fuel consumption is 30.02 mpg.

[Need more exercises?](#)

| 7.6 | Confidence Intervals for a Population Proportion |

To complete this section of homework watch Chapter Seven, Lecture Examples: **[113](#)**, **[114](#)**, *and* **[115](#)**.

54. Assume the given sample data below is being used to estimate the population proportion, and calculate the margin of error: VS

    The confidence level is 90% and out of 690 trials 298 were successful.

55. Assume the given sample data below is being used to estimate the population proportion, and calculate the margin of error:

    The confidence level is 95% and out of 1000 trials 450 were successful.

56. Assume the given sample data below is being used to estimate the population proportion, and calculate the margin of error: VS

    The confidence level is 98% and out of 800 trials 29% were successful.

57. Assume the given sample data below is being used to estimate the population proportion, and calculate the margin of error:

    The confidence level is 99% and out of 1235 trials 74% were successful.

58. Use the following data to form a confidence interval for the proportion: n = 3896, x = 702, Clevel = 95% VS

59. Use the following data to form a confidence interval for the proportion: n = 1896, x = 500, Clevel = 98%

60. Use the following data to form a confidence interval for the proportion: n = 8000, x = 5234, Clevel = 90%

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

61. Use the following data to form a confidence interval for the proportion: n = 506, x = 390, Clevel = 99%.

62. If you wanted to determine the number of people needed for a survey to determine the percentage of the population without health insurance, how could this be done? Assume you would like to ultimately form a 95% confidence interval with a margin of error of just 2 percentage points (hint: think of the margin of error formula for the proportion interval). VS 🖳

63. A recent survey of 1000 married men revealed that 56% of them have been unfaithful at least once. Form a 95% confidence interval to estimate the true proportion of married men who are unfaithful. 🖳 VS

64. A company claims that they have a method for preselecting your child's eye color. Out of 2000 couples trying to have a child with blue eyes, 510 of them had a baby with blue eyes after undergoing the procedure. The company claims this is a success, but a researcher pointed out that these couples had a 25% chance of having a blue-eyed baby simply due to the genes they already carried. Form a 98% confidence interval to estimate the true proportion of similar parents that would give birth to a blue-eyed child after using the company's services. Can we conclude the company's methods produce more than a 25% success rate?

65. A common statistic that is thrown around is that at least half of all new marriages will end in divorce. A researcher followed the marriages of 1500 couples for ten years. In that time, 800 of the couples divorced. Use the data to form a 90% confidence interval for the true rate of divorce among new marriages. Does this evidence support the idea that at least half of all new marriages end in divorce? 🖳 VS

66. Last semester after reviewing 3000 course evaluations, I noticed that 1632 of them stated they had an A in the class at the time of the evaluation (just a day or two before the final exam). Form a 95% confidence interval on the true proportion of students who say they have an A in their class at the time of evaluations. College records indicate that in those same classes only 12% of the students earn an A as their final grade and only another 20% will earn a B (that is 32% with an A or B by the end). If an A student would not do so poorly on their final to lower their overall grade below a B, what can you conclude about the truthfulness of students when they report that they have an A in the class at the time of the evaluation?

67. When working for the Brooklyn District Attorney, investigator Robert Burton analyzed the leading digits of amounts on checks from companies that were suspected of fraud. Among 784 checks, 61% had amounts with leading digits of 5. Construct a 99% confidence interval estimate of the proportion of checks having amounts with a leading digit of 5. When checks are issued in the normal course of honest transactions, it is expected that 7.9% of the checks will have amounts with leading digits of 5. What does this confidence interval suggest about the legitimacy of these checks?

🖳: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

68. In a recent study of a random selection of 200 students there were 5 grades of C-. Construct the 95% confidence interval for the true proportion of C- grades for Statistics classes like the ones these students were chosen from. If the class grades were normally distributed, we'd expect that around 5% of the grades would be in the C- range. Looking at the interval we just created, what do you think?

69. A public health organization is interested in estimating the proportion of smokers among the population in America in 2022. They collected a random sample of 1000 individuals and found that the sample proportion is 0.18. Calculate a 95% confidence interval for the true proportion of smokers among the population in America in 2022. The smoking rate used to be greater than 22%, does this interval support the claim that the rate of smoking has dropped?

7.6   Answers:

54. 0.031

55. 0.03083

56. 0.03732

57. 0.03215

58. [0.168, 0.192]

59. [0.240, 0.287]

60. [0.646, 0.663]

61. [0.723, 0.819]

62. You can use the ME formula $E = Z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}}$ and solve for n the formula becomes

$\left(\dfrac{Z_{\alpha/2}}{E}\right)^2 \hat{p}\hat{q} = n$ Plug in the numbers from the problem and always remember to round up on sample size calculations.

63. [0.529, 0.591]

64. [0.232, 0.278] Since 25% is inside this interval it is possible that the true proportion is only 25% which would mean their method is not effective. Therefore, there is not enough evidence to conclude that their method works.

65. [0.512, 0.555] Since this interval only contains values over 50% it seems that the interval provides evidence that at least half of all new marriages end in divorce.

: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

66. [0.526, 0.562] Since only 32% of the students actually earn an A or B it seems that there is an inconsistency in the interval and reality. Based on the students' response, the interval says that the true proportion of A's in stats classes is between 53% and 56%, but in reality it is only 12% A's and 20% B's. This means students are probably lying about their standing in the class. This shows the interval is only as good as the data used to create it.

67. [0.565, 0.655] Since this is so much higher than the 7.9% it is supposed to be we must conclude that these checks are fraudulent.

68. [0.003, 0.047] This shows the true proportion should be between 0.3% and 4.7%. This is not too far from the hypothesized value of 5%, but still 5% is outside of the interval. It might be that the distribution is not normally distributed.

69. [0.156, 0.204] We are 95% confident the true percentage of smokers is between 15.6% and 20.4%. The interval supports the idea that the rate of smoking has dropped.

Chapter 7   Mixed Review

70. A random sample of 250 voters were asked if they planned to vote in the next midterm elections. Using the results, researchers formed the following 95% confidence interval estimate of the proportion of voters who plan to vote: 0.38 ± .05. If a pundit claims that less than half of the public plans to vote in the coming midterm elections, does the interval support the pundit's statement? VS

71. A biologist wishes to estimate the average amount of nitrogen in a certain canal to within 4 ppm and with 90% reliability. Initial tests indicate that the standard deviation for nitrogen levels is 19 ppm. How many samples of canal water must be sampled for the biologist to get the information she desires?

72. A business owner randomly samples 25 days of sales. The data is used to create the following 99% interval which estimates the average daily sales total: $250 to $600. The business owner is not happy with the resulting interval because it is very wide. If the business owner decides she is going to collect new data and try again, what steps can she take to ensure the interval has a smaller margin of error?

73. A sportswriter is trying to estimate the average amount of stoppage time that is added to the end of MLS soccer games. The writer randomly samples 50 games. He finds that the average amount of stoppage time added to those games is 3.2 minutes. The population standard deviation for the data is 1.2 minutes. Using this data, construct a 99% confidence interval to estimate the mean amount of stoppage time added to the end of MLS games.

74. A professor wants to estimate the average number of hours his working students work per week. The professor sampled a random selection of 15 students who work. The average number of hours worked per week for the sample of students was 21.3 hours. The sample standard deviation was 5.1 hours. Form a 95% confidence interval to estimate the true mean number of hours worked per week for working students.

75. A random sample of 500 students were asked about the costs of traditional textbooks. Three hundred ninety-five of the students stated that traditional textbooks are too expensive. Using the data from the sample, construct a 98% interval estimate of the true proportion of students who believe traditional textbooks are too expensive.

76. In the past, the mean running time for a certain type of flashlight battery has been 8.5 hours. The manufacturer has introduced a change in the production method, which they hope has increased the mean running time. The mean running time for a random sample of 22 light bulbs was 8.65 hrs with a standard deviation of 0.4 hrs. Use this data to find the margin of error that would be used to construct a 98% confidence interval estimate of the true mean running time.

77. A chef wishes to estimate the average amount of time it takes to prepare for each dinner service. He wants to form a 95% interval estimate of the mean time to prepare. He believes the standard deviation is 16 minutes. How many dinner preps must he randomly sample and time to estimate the true mean time to within 5 minutes?

Chapter 7 Mixed Review   Answers:

70. The interval shows the proportion of people reporting that they plan to vote is some value between 33% and 43%, so the pundit seems to be correct because this entire range of values is less than 50%.

71. 62 samples of canal water

72. She can increase the sample size, and/or she can lower the confidence level.

73. 2.76 to 3.64 minutes

74. 18.48 to 24.12 hours per week

75. 74.8% to 83.2%

76. 0.215

77. 40 preps must be timed