ANOVA:
Comparing More Than Two Means

10.1 ANOVA: The Completely Randomized Design

To complete this section of homework watch Chapter Ten, Lecture Examples: 151 A, 151 B, 151tech, 152, and 152tech.

1. Clothing manufacturers use a wear-testing machine to measure different fabrics' ability to withstand abrasion. The wear of the material is measured by weighing the clothing after it has been through the wear-testing machine. A manufacturer wants to determine if there is a difference between the average weight loss among four different materials. The experiment is done by using four samples of each kind of material. The samples were tested in a completely randomized order. The weights are listed below. Use the data below and a 1% significance level to construct an ANOVA table and to determine if at least one fabric is significantly different from the others. VS Note: $\sum y_i^2 = 92.9719$

	Fabric			
	A	B	C	D
	1.93	2.55	2.40	2.33
	2.38	2.72	2.68	2.40
	2.20	2.75	2.31	2.28
	2.25	2.70	2.28	2.25
Totals	8.76	10.72	9.67	9.26

2. Clothing manufacturers use a wear-testing machine to measure different fabrics' ability to withstand abrasion. The wear of the material is measured by weighing the clothing after it has been through the wear-testing machine. A manufacturer wants to determine if there is a difference between the average weight loss among four different materials. The experiment is done by using eight samples of each kind of material. The samples were tested in a completely randomized order. The weights are listed below. Use the ANOVA display below and a 1% significance level determine if at least one fabric is significantly different from the others.

```
Factor   Type   Levels  Values
Fabric   fixed       4  A, B, C, D

Analysis of Variance for Weight
Source   DF       SS      MS     F      P
Fabric    3   5.7153  1.9051  6.46  0.002
Error    28   8.2605  0.2950
Total    31  13.9758

Pooled S = 0.543156
```

: indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com

a) What is the null and alternative hypotheses for this ANOVA procedure?
b) What is the p-value for this test?
c) What is the value of the test statistic for this test?
d) What is the decision regarding the null hypothesis? Justify this decision using the p-value from the test.
e) Based on the results of this experiment, do each of the four fabric types wear the same? In other words, do the fabrics seem to lose the same amount of material during the tests?
f) If the fabric types do not all wear the same, can we determine which fabrics differ from each other significantly using the provided results alone?

3. A study was conducted to determine the factor that reduces blood pressure the most: medication, diet, or exercise. Fifteen patients at a hospital with comparable levels of high blood pressure are randomly assigned to each treatment group. After eight weeks, the drop in systolic blood pressure for each patient was measured. Use the data below and a 5% significance level to construct an ANOVA table to test the claim that all three of the treatments produce the same drop in blood pressure. VS Note: $\sum y_i = 121$ and $\sum y_i^2 = 1{,}161$

Treatment		
Medication	Exercise	Diet
11	7	12
10	8	6
8	4	10
14	2	8
13	3	5

4. A study was conducted to determine the factor that reduces blood pressure the most: medication, diet, or exercise. Thirty patients at a hospital with comparable levels of high blood pressure are randomly assigned to each treatment group. After eight weeks, the drop in systolic blood pressure for each patient was measured. Use the computer output below and a 5% significance level to test the claim that all three of the treatments produce the same drop in blood pressure.

```
Factor      Type    Levels  Values
Treatment   fixed        3  M, E, D

Analysis of Variance for BP Reduction
Source      DF        SS        MS       F       P
Treatment    2   132.467    66.233    9.37   0.001
Error       27   190.900     7.070
Total       29   323.367

Pooled S = 2.65902
```

a) What is the null and alternative hypotheses for this ANOVA procedure?
b) What critical value would be used for the test of the null hypothesis?
c) Why is there a pooled standard deviation reported?

d) What is the connection between the pooled standard deviation and the provided MSE?
e) What is the p-value for this test?
f) What is the decision regarding the null hypothesis? Justify this decision using the p-value from the test.
g) Based on the results of this experiment, do each of the three treatments lower blood pressure equally on average?
h) If the treatments do not all have the same effect on blood pressure levels, can we determine which treatments differ from each other significantly using the provided results alone?

5. Glue Strength: Four adhesives that are used to fix porcelain to teeth are tested in a completely randomized design. The experiment bonds porcelain to teeth and then a machine is used to pry the tooth from the porcelain. The amount of force needed to do this for each bond is recorded. Use the results below and a 1% significance level to construct an ANOVA table to test the claim that there is a significant difference between the bonding strengths. VS

($\sum y_i = 5,572$ and $\sum y_i^2 = 1,324,754$)

	Adhesive			
	204	197	264	248
	181	223	226	138
	203	232	249	220
	262	207	255	304
	230	223	237	268
	288	197	240	276
Totals	**1368**	**1279**	**1471**	**1454**

6. Glue Strength: Four adhesives that are used to fix porcelain to teeth are tested in a completely randomized design. The experiment bonds porcelain to teeth and then a machine is used to pry the tooth from the porcelain. The amount of force needed to do this for each bond is recorded. Complete the ANOVA table below and use a 1% significance level to test the claim that there is a significant difference between the bonding strengths.

```
Factor  Type    Levels  Values
Glue    fixed        4  A, B, C, D

Analysis of Variance for Force
Source   DF       SS     MS      F    Sig.
Glue            3884                 0.476
Error
Total    39    58766

Pooled S = ?
```

a) Fill in the missing parts of the given ANOVA table.
b) What is the null hypothesis and alternative hypothesis this ANOVA procedure is testing?
c) What **degrees of freedom** for the critical value would be used for the test of the null hypothesis?
d) What was the value of the pooled standard deviation used during this procedure?

e) What is the p-value for this test?
f) What is the decision regarding the null hypothesis? Justify this decision using the p-value from the test.
g) Based on the results of this experiment, do each of the four adhesives have the same bonding strength?

7. The time it takes for three brands of caulk to fully dry is studied by a construction contractor. Six beads from each of the three brands are randomly placed in the same space in a bathroom. The time for each bead to dry is listed below. Form an ANOVA table and use a 5% significance level to test the claim there is a difference between the drying times for the different brands of caulk.

VS $\sum y_i = 438.8$ $\sum y_i^2 = 10{,}772.4$

	Brand		
	A	B	C
	24.7	22.1	25.7
	28.6	20.2	24.3
	25.1	21.1	23.6
	25.3	23.5	26.1
	26.0	22.8	26.9
	25.9	22.7	24.2
Totals	**155.6**	**132.4**	**150.8**

8. The time it takes for three brands of caulk to fully dry is studied by a construction contractor. Eleven beads from each of the three brands are randomly placed in the same space in a bathroom. The time for each bead to dry is listed below. Complete the provided ANOVA table and use a 5% significance level to test the claim there is a difference between the drying times for the different brands of caulk.

```
Factor  Type   Levels  Values
Caulk   fixed       3  A, B, C

Analysis of Variance for Time
Source   DF        SS       MS      F   Sig.
Caulk                                   0.000
Error          44.464
Total         109.530

Pooled S = ?
```

a) Fill in the missing parts of the given ANOVA table.
b) What is the null hypothesis this ANOVA procedure is testing?
c) What critical value would be used for the test of the null hypothesis?
d) What was the value of the pooled standard deviation used during this procedure?
e) What is the p-value for this test?
f) What is the decision regarding the null hypothesis? Justify this decision using the p-value from the test.
g) Based on the results of this experiment, do each of the three caulks take the same length of time to dry?

h) If the caulks do not all have the same average drying time, can we determine which caulks differ significantly with regard to the time it takes for them to dry?

9. The table below provides the number of pounds lost by 16 different athletes who were each randomly assigned to one of four weight loss treatments (three different weight loss supplements and a placebo). Use the data below and a 2.5% significance level to test for differences between the four treatments. Do these supplements seem to work? Form an ANOVA table as part of the test. VS $\sum y_i^2 = 723$

Treatment			
Placebo	SupA	SupB	SupC
1	3	10	8
4	6	11	3
2	7	14	2
3	4	8	5

10. Researchers recorded the number of pounds lost by 32 different athletes who were each randomly assigned to one of four weight loss treatments (three different weight loss supplements and a placebo). Complete the ANOVA table below and use a 2.5% significance level to test for differences between the four treatments.

```
Factor       Type   Levels  Values
Supplement   fixed       4  A, B, C, P

Analysis of Variance for Weight Loss
Source       DF       SS      MS      F   Sig.
Supplement          1.203                 0.790
Error
Total              33.392

Pooled S = ?
```

a) Fill in the missing parts of the given ANOVA table.
b) What is the null hypothesis and alternative hypothesis this ANOVA procedure is testing?
c) What critical value would be used for the test of the null hypothesis?
d) What was the value of the pooled standard deviation used during this procedure?
e) What is the p-value for this test?
i) What is the decision regarding the null hypothesis? Justify this decision using the test statistic and critical value for the test.
f) Based on the results of this experiment, do each of the four weight loss treatments produce the same average weight loss?
g) Does the conclusion of this test indicate that some or all of these tested supplements work better than a sugar pill?

11. Orange trees at a citrus farm near Orlando were randomly assigned to one of three new fertilizers or the traditional fertilizer being used already. The new fertilizers are supposed to produce heavier oranges. At the 2.5% significance level, test the claim that at least one of the new fertilizers produce a heavier orange. Form an ANOVA table as part of the test. $\sum y_i = 4,228$ $\sum y_i^2 = 772,746$

	Fertilizers			
	Old	New A	New B	New C
	123	201	144	220
	145	200	146	217
	201	210	165	210
	154	185	192	106
	127	190	154	234
	168	168	153	215
Totals	**918**	**1154**	**954**	**1202**

12. Orange trees at a citrus farm near Orlando were randomly assigned to one of three new fertilizers or the traditional fertilizer being used already. The new fertilizers are supposed to produce heavier oranges. At the 2.5% significance level, test the claim that at least one of the new fertilizers produce a heavier orange.

```
Factor       Type   Levels  Values
Fertilizer   fixed       4  A, B, C, O

Analysis of Variance for Weight
Source       DF        SS      MS      F       P
Fertilizer          1202.8                 0.427
Error
Total        43   18124.8

Pooled S = ?
```

a) Fill in the missing parts of the given ANOVA table.
b) What are the hypotheses this ANOVA procedure is testing?
c) What critical value would be used for the test of the null hypothesis?
d) What was the value of the pooled standard deviation used during this procedure?
e) What is the p-value for this test?
f) What is the decision regarding the null hypothesis? Justify this decision using the test statistic and critical value for the test.
g) Based on the results of this experiment, do each of the four fertilizers produce oranges that have the same average weight?

13. Eighteen identical batteries were randomly assigned to operate in different temperatures to see if there were any differences in their lifetimes under the different conditions. Use the table below to determine if temperature effects the lifetime of batteries (in hours). Form an ANOVA table as part of the test. $\sum y_i = 478.8$ and $\sum y_i^2 = 14,741$

	Temp (Fahrenheit)		
	$40°$	$80°$	$120°$
	36.3	*32.2*	*12.3*
	42.2	*31.6*	*15.2*
	30.1	*23.4*	*16.0*
	38.2	*28.9*	*15.9*
	40.0	*27.7*	*13.1*
	41.6	*24.7*	*9.4*
Totals	**228.4**	**168.5**	**81.9**

14. Complete the ANOVA table below for a CRD experiment: VS

Source	df	SS	MS	F
Treatment	3	1843.67		
Error				
Total	23	1947.34		

15. Complete the ANOVA table below for a CRD experiment: VS

Source	df	SS	MS	F
Treatment	2			
Error	12	23.572		
Total		99.693		

16. Consider the ANOVA table below for a CRD experiment. What do you conclude about the claim that the treatment means are all equal? VS

Source	df	SS	MS	F
Fertilizers	2	1096	548	
Error	24	1440	60	
Total	26	2536		

10.1 Answers

1. $H_0 : \mu_A = \mu_B = \mu_C = \mu_D$
   H_A : At least one mean significantly differs from the others

Source	Df	SS	MS	F	
Material	3	0.520	0.173	8.534	
Error	12	0.244	0.020		
Total	15	0.764			

Critical Value: 5.953

Since F > Critical value, reject the null and support the alternative. At least one pair of the fabrics wears differently.

2.
   a. $H_0 : \mu_A = \mu_B = \mu_C = \mu_D$
      H_A : At least one mean significantly differs from the others
   b. p-value = 0.002
   c. Test statistic: F = 6.46
   d. Reject the null hypothesis since the p-value is smaller than alpha (0.002 < 0.01)
   e. Since we rejected the null hypothesis that says they all wear the same, the fabrics do not seem to wear the same.
   f. It is not possible to determine which fabrics differ significantly from each other. We would need a multiple comparison procedure, which is covered in the next section.

3.

$H_0 : \mu_M = \mu_E = \mu_D$
H_A : At least one mean significantly differs from the others

Source	df	SS	MS	F	
Treatment	2	102.53	51.267	7.466	
Error	12	82.4	6.867		
Total	14	184.93			

Critical Value: 3.8853; Since F > Critical value, reject the null and support the alternative. The sample data allows us to reject the claim that all the treatments produce the same drop in blood pressure. In other words, at least one pair of the treatments reduce blood pressure differently.

4. Solution:
   a. $H_0 : \mu_M = \mu_E = \mu_D$
      H_A : At least one mean significantly differs from the others
   b. $F_{2,27,0.05} = 3.3541$
   c. Part of the assumptions for the ANOVA CRD experiment is that all of the k samples come from populations with the same variance (and thus they have the same standard deviations), so Minitab has provided an estimate of the common standard deviation.
   d. The pooled S is the same as the square root of the MSE. In other words, the MSE is an estimate of the pooled variance. $S^2_{pooled} = MSE$
   e. p -value = 0.001
   f. Reject the null hypothesis since the p-value is smaller than alpha (0.001 < 0.05)
   g. Since we rejected the null hypothesis that says they all reduce BP the same on average, the treatments do not seem to work the same.
   h. It is not possible to determine which treatments differ significantly from each other. We would need a multiple comparison procedure, which is covered in the next section.

5.

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_A : At least one mean significantly differs from the others.

Source	df	SS	MS	F	
Adhesive	3	3904.33	1301.44	0.956	
Error	20	27217	1360.85		
Total	23	31121.33			

Critical Value: 4.938; Since F < Critical value, do not reject the null and do not support the alternative. The sample data does not support the claim that at least one pair of the adhesives bond differently.

6. Solution:

```
a. Factor  Type   Levels  Values
   Glue    fixed       4  A, B, C, D

   Analysis of Variance for Force
   Source  DF     SS     MS      F   Sig.
   Glue     3   3884 1294.7 0.849 0.476
   Error   36  54882 1524.5
   Total   39  58766
```

```
Pooled S = 39.045
```

b. $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_A : At least one mean significantly differs from the others.

c. Three for the numerator and 36 for the denominator.

d. Pooled S = 39.045

e. P-value = 0.476

f. Do not reject the null hypothesis since the p-value is larger than alpha (0.476 > 0.01)

g. It seems they all have the same strength, because we cannot reject the idea that each of the four adhesives bond with the same strength.

7. $H_0 : \mu_A = \mu_B = \mu_C$
H_A : At least one mean significantly differs from the others

Source	df	SS	MS	F	
Treatment	2	49.991	24.996	14.738	
Error	15	25.44	1.696		
Total	17	75.431			

Critical Value: 3.6823; Since F > Critical value, reject the null and support the alternative. The sample data allows us to support the claim that at least one pair of the caulk brands dry at different rates.

8. a.

```
Factor  Type   Levels  Values
Caulk   fixed       3  A, B, C

Analysis of Variance for Time
Source  DF       SS      MS       F   Sig.
Caulk    2   65.066  32.533  21.950  0.000
Error   30   44.464   1.482
Total   32  109.530

Pooled S = 1.2174
```

b. $H_0 : \mu_A = \mu_B = \mu_C$
H_A : At least one mean significantly differs from the others

c. $F_{2,30,0.05} = 3.3158$

d. Pooled S = 1.2174

e. P-value = 0.000

f. Reject the null hypothesis since the p-value is smaller than alpha (0.000 < 0.05)

g. Since we rejected the null hypothesis that says they all take the same time to dry on average, the caulks do not seem to dry at the same speed.

h. It is not possible to determine which caulks differ significantly from each other using this alone. We would need a multiple comparison procedure, which is covered in the next section.

9. $H_0: \mu_A = \mu_B = \mu_C = \mu_P$
H_A: At least one mean significantly differs from the others

Source	df	SS	MS	F	
Material	3	150.69	50.23	11.009	
Error	12	54.75	4.5625		
Total	15	205.44			

Critical Value: 4.4742; Since F > Critical value, reject the null and support the alternative. The sample data supports the claim that at least one pair of the weight loss supplements reduce weight in different average amounts. Yes, it seems that at least one of the supplements works.

10. a.

```
Factor      Type   Levels  Values
Supplement  fixed       4  A, B, C, P

Analysis of Variance for Weight Loss
Source       DF      SS      MS     F   Sig.
Supplement   3    1.203  0.401 0.349  0.790
Error       28   32.189 1.1496
Total       31   33.392

Pooled S = 1.0722
```

b. $H_0: \mu_A = \mu_B = \mu_C = \mu_P$
H_A: At least one mean significantly differs from the others

c. $F_{3,28,0.025} = 3.6264$

d. Pooled S = 1.0722

e. P-value = 0.790

f. Do not reject the null hypothesis since the F-test stat, 0.3488 is less than the critical value 3.6264. We only reject if the test stat is larger than the critical value.

g. It seems they all work the same, because we cannot reject the null hypothesis that says the means are all equal.

h. Since the experiment included a placebo (a sugar pill), it seems each of the supplements work no better than a sugar pill. If this is in fact reality, the only benefit to the supplements would be the placebo effect.

11. $H_0: \mu_A = \mu_B = \mu_C = \mu_O$
H_A: At least one mean significantly differs from the others

Source	df	SS	MS	F	
Material	3	10060.67	3353.56	3.757	
Error	20	17852.67	892.63		
Total	23	27913.34			

Critical Value: 3.8587; Since F < Critical value, Do not reject the null and do not support the alternative. The sample data does not support the claim that at least one of the fertilizers produces a heavier orange.

12. a.

```
Factor       Type   Levels  Values
Fertilizer   fixed       4  A, B, C, O

Analysis of Variance for Weight
Source       DF       SS     MS     F      P
Fertilizer    3   1202.8 400.93 0.948  0.427
Error        40  16922.0 423.05
Total        43  18124.8

Pooled S = 20.568
```

b. $H_0 : \mu_A = \mu_B = \mu_C = \mu_O$
H_A : At least one mean significantly differs from the others

c. $F_{3,40,0.025} = 3.4633$

d. Pooled S = 20.568

e. P-value = 0.427

f. Do not reject the null hypothesis since the F-test stat, 0.948 is less than the critical value 3.4633. We only reject if the test stat is larger than the critical value.

g. It seems all the fertilizers work the same, because we cannot reject the null hypothesis that says the means are all equal. This means the original fertilizer is just as effective as the new ones.

13. $H_0 : \mu_{40} = \mu_{80} = \mu_{120}$
H_A : At least one mean significantly differs from the others

Source	df	SS	MS	F	
Treatment	2	1808.32	904.16	68.986	
Error	15	196.60	13.11		
Total	17	2004.92			

Critical Value: 3.6823 (using 5% as a default alpha value); Since F > Critical value, reject the null and support the alternative. The sample data allows us to support the claim that at least one temperature effects the lifetime of a battery.

14.

Source	df	SS	MS	F
Treatment	3	1843.67	614.56	118.56
Error	20	103.67	5.1835	
Total	23	1947.34		

15.

Source	df	SS	MS	F
Treatment	2	76.121	38.061	19.376
Error	12	23.572	1.964	
Total	14	99.693		

16. F = 9.133, Critical Value: $F_{2,24,0.05} = 3.4028$; Reject the null; The sample data allows us to reject the claim…

Need more exercises?

10.2 Multiple Comparisons of Means

***To complete this section of homework watch Chapter Ten, Lecture Examples 153, 154, 155, 156, 157A, 157B, 157.4, 157.5,* and *157.6*.**

17. Confidence intervals for the difference between two means $\mu_1 - \mu_2$ are provided below. For each part, which mean is significantly larger? A. (-1.25, -0.52) B. (8.3, 11.6) C. (-2.3, 4.6) VS

18. If a completely randomized design experiment involves k treatments, for each part below, determine the number of comparisons of treatment means that will need to be made in a multiple comparison procedure. A. k = 3 B. k = 5 C. k = 8 D. k = 11 VS

: indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com

19. A multiple comparison procedure for comparing four treatment means produced the confidence intervals shown below. Rank the means from smallest to largest. Indicate which means are significantly different. VS

$$\mu_1 - \mu_2 : (2,15)$$
$$\mu_1 - \mu_3 : (4,7)$$
$$\mu_1 - \mu_4 : (-10,3)$$
$$\mu_2 - \mu_3 : (-5,11)$$
$$\mu_2 - \mu_4 : (-12,-6)$$
$$\mu_3 - \mu_4 : (-8,-5)$$

20. A multiple comparison procedure for comparing four treatment means produced the confidence intervals shown below. Rank the means from smallest to largest. Indicate which means are significantly different. VS

$$\mu_A - \mu_B : (-8,2)$$
$$\mu_A - \mu_C : (3,6)$$
$$\mu_A - \mu_D : (14,17)$$
$$\mu_B - \mu_C : (5,15)$$
$$\mu_B - \mu_D : (6,11)$$
$$\mu_C - \mu_D : (-2,9)$$

21. A multiple comparison procedure is conducted and the results are given below. The means have been ranked from smallest to largest. Interpret the results and state the number of comparisons made. $\overline{DB}AC$ VS

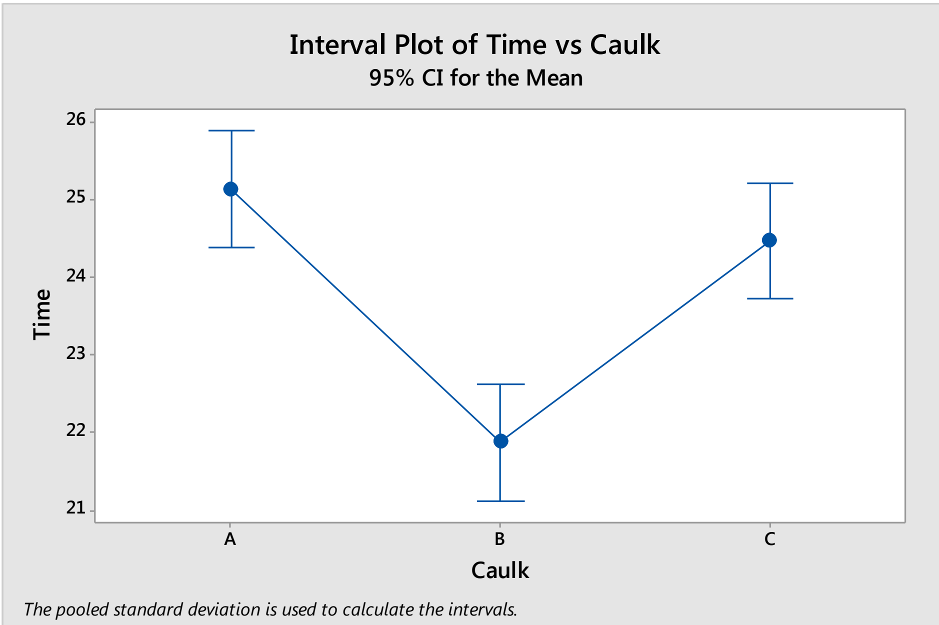
22. In a CRD experiment at a significance level of 5% with a balanced design (i.e.- all the treatment sample sizes are the same) the F statistic turns out to be 2.29 which has a p-value of 0.1769. What multiple comparison procedure should be used? VS

23. In a CRD experiment at a significance level of 5% with a balanced design (i.e.- all the treatment sample sizes are the same) the F statistic turns out to be 14.40 which has a p-value of 0.0038. What multiple comparison procedure should be used? VS

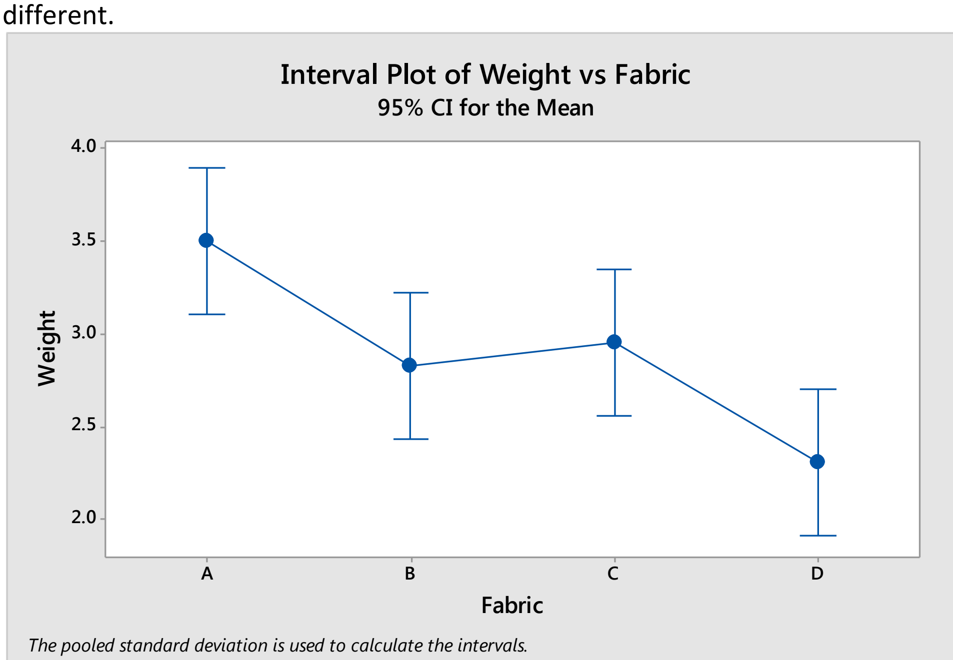
24. In a CRD experiment at a significance level of 1% with an unbalanced design (i.e.- all the treatment sample sizes are **not** the same) the F statistic turns out to be 11.42 which has a p-value of 0.0068. What multiple comparison procedure should be used? VS

25. Which multiple comparison procedure produces the shortest interval widths? The second shortest interval widths? The widest interval widths?

26. The time it takes for three brands of caulk to fully dry was studied by a construction contractor. Eleven beads from each of the three brands were randomly placed in the same space in a bathroom and then timed as they dried. Using the provided graph below, rank the means from smallest to largest, and indicate which means are significantly different.



27. Clothing manufacturers use a wear-testing machine to measure different fabrics' ability to withstand abrasion. The wear of the material is measured by weighing the clothing after it has been through the wear-testing machine. A manufacturer wants to determine which material loses the least amount of weight among the four tested materials. Using the provided graph below, rank the means from smallest to largest, and indicate which means are significantly different.



10.2 Answers

17. A. mean 2 is larger B. Mean 1 is larger C. The means are not significantly different.
18. A. 3 B. 10 C. 28 D. 55
19. $\overline{3\;2}\;\overline{1\;4}$
20. $\overline{\mathrm{D\;C}}\;\overline{\mathrm{A\;B}}$
21. 6 comparisons were made, and the results are as follows: C is significantly larger than every other mean. A is significantly larger than B and D. B's sample mean was larger than D's, but not significantly.
22. You do not need to make multiple comparisons because with a p-value of 0.1769 the null cannot be rejected at a 5% significance level. This means that we cannot say a significant difference exists among the means, so there is no need to rank them.
23. Tukey is the best option for a balanced design.
24. Bonferroni is the best option for an unbalanced design.
25.

Tukey ---------Equal Pairwise (shortest)

Bonferroni ----Equal or Unequal Pairwise (2nd shortest)

Scheffe` ------Equal or Unequal General Contrasts (widest)

26. $B\overline{CA}$
27. $\overline{D\overline{BC}}\overline{A}$

Need more exercises?

10.3 ANOVA: The Randomized Block Design

To complete this section of homework watch Chapter Ten, Lecture Examples 158 A, 158 B, 158tech, 159, and 159tech.

28. To test tire tread loss for four different brands of tires, researchers put the four tires on six different cars. In order to avoid the confounding of driver differences, car differences, and tire differences, they decide to put one tire of each brand onto each car. This way, each car has all four tires on it. The four different tires are then placed on each car in random order. Each car will be driven for 30,000 miles and then the tires will be measured for tread thickness. The loss in thickness will be recorded. Computer output has provided a partial ANOVA table for the RBD experiment. Complete the ANOVA table and answer the questions that follow:

Factor Information

Factor	Levels	Values
Tire Brand	4	A, B, C, D
Car	6	1, 2, 3, 4, 5, 6

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Tire Brand	?	174.167	?	?	<0.0001
Car	?	11.000	2.2000	?	0.6709
Error	?	?	?		
Total	23	236.500			

a. What is the null hypothesis for this ANOVA procedure to test if the tire brands wear differently?
b. What is the p-value for this test?
c. What is the value of the test statistic for this test?
d. At the 1% significance level, what is the decision regarding the null hypothesis? Justify this decision using the p-value from the test.
e. Based on the results of this experiment, do each of the four tire brands wear the same?

29. To test tire tread loss for four different brands of tires, researchers put the four tires on four different cars. In order to avoid the confounding of driver differences, car differences, and tire differences, they decide to put one tire of each brand onto each car. This way, each car has all four tires on it. The four different tires are then placed on each car in random order. Each car will be driven for 30,000 miles and then the tires will be measured for tread thickness. The loss in thickness will be recorded. The data for this RBD experiment is included below. At the 5% significance level, test the claim that the tires do not all wear the same.

Car	Brand Tires				Totals
	A	B	C	D	
I	17	14	12	13	56
II	14	14	12	11	51
III	13	13	10	11	47
IV	13	8	9	9	39
Totals	57	49	43	44	193
$\sum y^2$	823	625	469	492	2409

30. Complete the following ANOVA table, which was produced by a randomized block design experiment, then use it to answer the following questions:

Source	df	SS	MS	F
Treatments	4	501		
Blocks	2	225		
Error	8	110		
Total	14			

a. How many blocks and treatments were used in the experiment?

b. How many observations were made in the experiment?

c. What null and alternative hypothesis would be used in comparing the treatment means?

d. What test stat would be used when comparing the treatment means?

e. What would the rejection region be for the test?

f. What is the conclusion for the hypothesis test comparing the treatment means?

31. The effects of four types of graphite coater on light-box readings are to be studied. Since readings will differ from day to day, observations are taken on each of the four types every day. The results are as follows: VS

	Graphite coater type			
Day	M	A	K	L
1	4	4.8	5	4.6
2	4.8	5	5.2	4.6
3	4	4.8	5.6	5

(Note: $SS_{total} = 2.2766667$ and $\sum y_i = 57.4$)

Create the ANOVA table for this data and at the 5% level test the claim that all of the graphite coaters produce the same average light-box readings.

32. Test anxiety can hinder academic performance, so a researcher wants to compare the effectiveness of three treatments to reduce test anxiety. The procedure is used on 7 different students. Complete the provided ANOVA display and answer the following questions:

Factor Information

Factor	Levels	Values
Treatment	3	BetaBlocker, Meditation, ValerianRoot
Subject	7	1, 2, 3, 4, 5, 6, 7

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Treatment	?	0.4610	?	?	0.8308
Subject	?	25.2362	?	?	0.0327
Error	12	?	?		
Total	?	40.3895			

a. What is the null hypothesis for this ANOVA procedure to test if the treatments affect anxiety?
b. What is the critical value for the test of the treatment effect (use $\alpha = 0.05$)?
c. What is the value of the test statistic for the test of the treatment effect?
d. What is the p-value for the test of the treatment effect?
e. At the 5% significance level, can we reject the claim that the three different treatments reduce anxiety equally? Justify your answer using the critical value method.
f. At the 5% significance level, can we conclude that different subjects experience anxiety at different levels? Justify this decision using the appropriate p-value from the ANOVA display.

33. Test anxiety can hinder academic performance, so a researcher wants to compare the effectiveness of three treatments to reduce test anxiety. The procedure is used on 5 different students. Use the resulting data below and a 1% significance level to test the claim that the three different methods reduce anxiety equally. VS

	Anxiety level on a visual-analogue scale			
Subject	Beta Blocker	Valerian Root	Meditation	**Totals**
1	2.7	1.3	1	5
2	3.9	3.6	3.1	10.6
3	4.1	4.2	3.9	12.2
4	4.3	4.1	4	12.4
5	2.9	2.8	2.2	7.9
Totals				

(Note: $SS_{total} = 15.569333$ and $\sum y_i = 48.1$)

34. Grocery costs vary for different families, but a researcher wants to study the weekly cost of groceries for typical Florida families at four different grocery chains in South Florida. To do this,

the researcher looks at weekly costs for groceries at the four stores for seven different families. Each family will be randomly assigned to visit a different store each week, and each family will visit each store only once during the month long study period. Complete the provided ANOVA display and answer the questions that follow:

Factor Information

Factor	Levels	Values
Store	4	Costco, Publix, Target, WholeFoods
Family	7	1, 2, 3, 4, 5, 6, 7

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Store	?	?	?	?	<0.0001
Family	?	5863.5	?	?	0.1814
Error	?	?	611.20		
Total	?	43888.6			

a. What is the null hypothesis for this ANOVA procedure to test the claim that the four stores have different average grocery prices?
b. What is the critical value for the test of the store effect (use $\alpha = 0.025$)?
c. What is the value of the test statistic for the test of the store effect?
d. What is the p-value for the test of the store effect?
e. At the 2.5% significance level, can we support the claim that the four different stores have different average prices? Justify your answer using either the p-value method or the critical value method.

35. Grocery costs vary for different families, but a researcher wants to study the weekly cost of groceries for typical Florida families at four different grocery chains in South Florida. To do this, the researcher looks at weekly costs for groceries at the four stores for four different families. Each family will visit a different one of the four stores to shop each week for a month. The families will randomly be assigned to the stores each week. Use the data below and a 2.5% significance level to test the claim that the four stores have different average grocery prices.
VS

	Store			
Family	Publix	Target	Costco	Whole Foods
1	210	195	200	315
2	300	250	275	400
3	176	171	189	223
4	148	127	130	162

(Note: $SS_{total} = 83328.9375$ and $\sum y_i = 3471$)

: indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com

36. Complete the following ANOVA display and answer the following questions:

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Treatment	3	6172.2	?	?	0.0007
Block	4	?	?	1.55	0.2505
Error	?	?	173.11		
Total	19	9321.8			

a. How many treatments did this RBD experiment involve?
b. How many blocks did this RBD experiment involve?
c. How many total measurements were made during the experiment (n = ?)?
d. Was the treatment effect significant?
e. Were the blocks significant?

37. Calculate the SS_{total} for the following RBD data: VS

	Scale			
Inspector	I	II	III	
1	16	10	11	
2	15	9	14	
3	13	11	13	

10.3 Answers

28. Solution:

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Tire Brand	3	174.167	58.056	16.965	<0.0001
Car	5	11.000	2.2000	0.643	0.6709
Error	15	51.333	3.4222		
Total	23	236.500			

a. $H_0 : \mu_A = \mu_B = \mu_C = \mu_D$
b. the p-value is less than 0.0001
c. F = 16.965
d. Reject the null hypothesis since the p-value is smaller than alpha (0.000 < 0.01)
e. Since we rejected the null hypothesis that says they all wear same on average, the tires do not seem to wear at the same rate.

: indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com

29. ANOVA table

CF = 2328.0625, $SS_{total} = 2409 - CF = 80.9375$, SST = 2358.75 – CF = 30.6875,

SSB = 2366.75 – CF = 38.6875, SSE = 80.9375 – 30.6875 – 38.6875

Source	df	SS	MS	F
Tires	3	30.6875	10.22917	7.962
Car	3	38.6875	12.89583	10.038
Error	9	11.5625	1.284722	
Total	15	80.9375		

Critical Value: $F_{3,9,0.05} = 3.8625$

Since the test stat for tires is in the rejection region, we can reject the null and support the claim that the tires do not all wear the same.

30. Solution:

a) B = 3, t = 5

b) 15

c) $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$
H_A : At least one mean significantly differs from the others.

d) $\frac{MST}{MSE} = \frac{125.25}{13.75} = 9.109$

e) $F_{4,8,0.05} = 3.8379$

f) At least two means differ significantly.

31. ANOVA table

CF = 274.563, $SST = \frac{12.8^2}{3} + \frac{14.6^2}{3} + \frac{15.8^2}{3} + \frac{14.2^2}{3} - CF = 1.5303$

$$SSB = \frac{18.4^2}{4} + \frac{19.6^2}{4} + \frac{19.4^2}{4} - CF = 0.207$$

Source	df	SS	MS	F
Coater	3	1.5303	0.5101	5.674
Day	2	0.207	0.1035	1.151
Error	6	0.5394	0.0899	
Total	11	2.276667		

Critical Value: $F_{3,6,0.05} = 4.7571$

Since the test stat for coatings is in the rejection region, we can reject the null and support the alternative hypothesis. The coaters do not seem to all have the same light-box readings.

32. Solution:

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Treatment	2	0.4610	0.2305	0.188	0.8308
Subject	6	25.2362	4.2060	3.435	0.0327
Error	12	14.6923	1.2244		
Total	20	40.3895			

a. $H_0 : \mu_B = \mu_M = \mu_V$
b. $F_{2,12,0.05} = 3.8853$
c. F = 0.188
d. p-value = 0.8308
e. Do not reject the null hypothesis since the F-test stat, 0.188 is less than the critical value 3.8853. We only reject if the test stat is larger than the critical value. It seems they all work the same, because we cannot reject the null hypothesis that says the means are all equal.
f. Yes, it appears that individual subjects have different levels of anxiety, since we can reject the null hypothesis that says they experience the same levels on average (p-value 0.0327 < alpha 0.05).

33. ANOVA table
CF = 154.241

Source	df	SS	MS	F
Treatment	2	1.369	0.685	5.566
Subject	4	13.216	3.304	26.862
Error	8	0.984	0.123	
Total	14	15.569		

Critical Value: $F_{2,8,0.01} = 8.649$

Since the test stat for treatments is not in the rejection region, we cannot reject the claim that the three different methods reduce anxiety equally.

34. Solution:

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Store	3	27023.53	9007.843	14.738	<0.0001
Family	6	5863.5	977.25	1.599	0.1814
Error	18	11001.57	611.20		
Total	27	43888.6			

a. $H_0 : \mu_C = \mu_P = \mu_T = \mu_W$

b. $F_{3,18,0.025} = 3.9539$

c. F = 14.738

d. The p-value is less than 0.0001

e. Reject the null hypothesis since the p-value is smaller than alpha (0.000 < 0.025) or because the test stat is larger than the critical value (14.738 > 3.9539). Yes, it appears the stores have different average prices.

35. ANOVA table

CF = 752990.0625

Source	df	SS	MS	F
Store	3	19020.188	6340.063	9.278
Family	3	58158.688	19386.229	28.370
Error	9	6150.063	683.340	
Total	15	83328.938		

Critical Value: $F_{3,9,0.025} = 5.0781$

Since the test stat for store is in the rejection region, we reject the null and support the claim that at least two of the stores have different prices.

36. Solution:

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Treatment	3	6172.2	2057.4	11.885	0.0007
Block	4	1072.28	268.07	1.55	0.2505
Error	12	2077.32	173.11		
Total	19	9321.8			

a. T = 4

b. B = 5

c. n = 20

d. Yes, since the p-value is very small.

e. No, the p-value is large.

37. Solution:

$$CF = 1393.7777$$

$$\sum y_i^2 = 1438$$

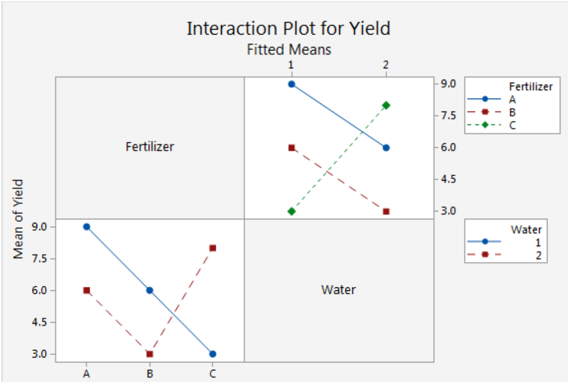
$$SS_{total} = 1438 - CF = 44.22223$$

Need more exercises?

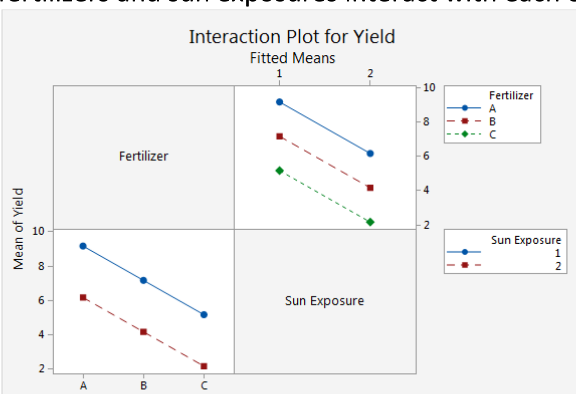
10.4 ANOVA: Factorial Experiments

To complete this section of homework watch Chapter Ten, Lecture Examples 159.1 and 159.2.

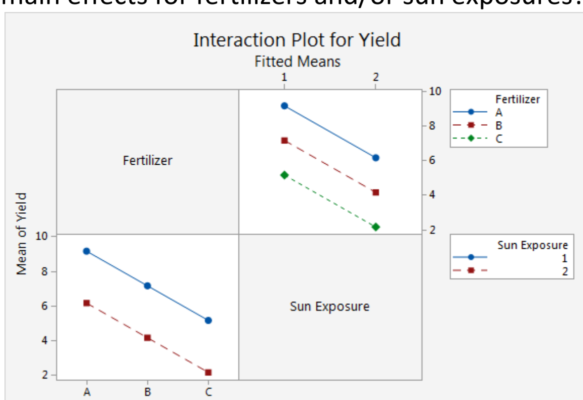
38. Consider the graph below of the mean yields for a 3 X 2 factorial experiment, which featured three different fertilizers and two different watering schedules. Do the graphs indicate that the fertilizers and watering schedules interact with each other?



39. Consider the graph below of the mean yields for a 3 X 2 factorial experiment, which featured three different fertilizers and two different sun exposures. Do the graphs indicate that the fertilizers and sun exposures interact with each other?

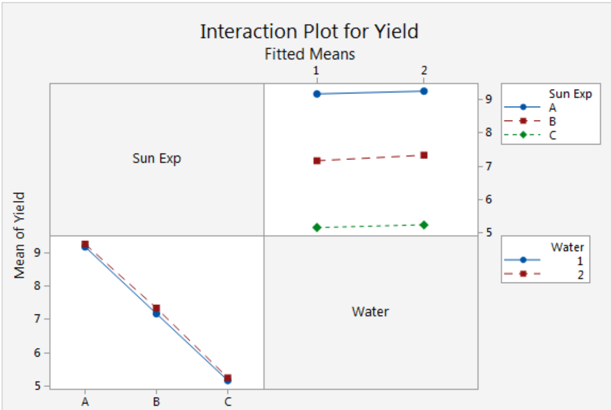


40. Consider the graph below of the mean yields for a 3 X 2 factorial experiment, which featured three different fertilizers and two different sun exposures. Do the graphs indicate significant main effects for fertilizers and/or sun exposures?

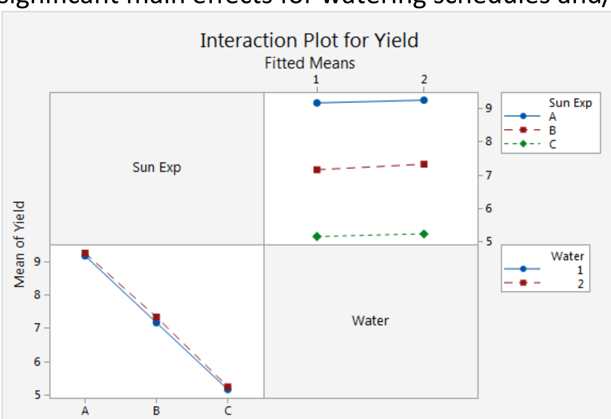


: *indicates the exercise has a video devoted to it in the corresponding section of STATSprofessor.com*

41. Consider the graph below of the mean yields for a 3 X 2 factorial experiment, which featured three different sun exposures and two different watering schedules. Do the graphs indicate that the sun exposures and watering schedules interact with each other?



42. Consider the graph below of the mean yields for a 3 X 2 factorial experiment, which featured three different sun exposures and two different watering schedules. Do the graphs indicate significant main effects for watering schedules and/or sun exposures?



43. Consider the ANOVA table below for a 3 X 2 factorial experiment on factors affecting sunflower crop yields, which featured three different fertilizers and two different watering schedules. Complete the ANOVA table and answer the questions that follow.

Factor Information

Factor	Levels	Values
Fertilizer	3	A, B, C
Water	2	1, 2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Fertilizer	2	12.5903	?	?	0.0017
Water	1	0.2222	0.2222	0.40	0.5390
Fertilizer*Water	?	?	?	?	<0.0001
Error	?	?	0.5556		
Total	17	44.0694			

a) Complete the missing parts of the ANOVA table above.

b) Identify the factors and levels for this experiment.

c) This two-factor factorial experiment can be referred to as a 3 X 2. Where does the 3 X 2 come from?
d) Give an example of a treatment for this experiment. How many different treatments are there?
e) How many replications were used for this experiment? Why is it necessary to have more than one?
f) What is the p-value for the F test statistic related to the interaction effect? What should we conclude about the interaction between these factors?
g) Based on the results of the test for an interaction effect, is it appropriate to test for main effects?
h) What is the next step in the analysis of this experiment's data?

44. Below is a partial ANOVA table below for a 2 X 3 factorial experiment conducted by a pickle manufacturer to determine the factors affecting Alum production, which featured three different raw materials and two different agitation speeds. Complete the ANOVA table and answer the questions that follow.

Factor Information

Factor	Levels	Values
Material	3	A, B, C
Speed	2	1, 2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Material	?	?	?	?	<0.0001
Speed	?	0.2222	0.2222	3.05	0.1064
Material*Speed	?	0.3611	0.1806	2.48	0.1258
Error	?	0.8750	?		
Total	17	47.4861			

a) Complete the missing parts of the ANOVA table above.
b) Identify the factors and levels for this experiment.
c) Give an example of a treatment for this experiment. How many different treatments are there?
d) How many replications were used for this experiment? Why is it necessary to have more than one?
e) What is the p-value for the F test statistic related to the interaction effect? What should we conclude about the interaction between these factors?
f) Based on the results of the test for an interaction effect, is it appropriate to test for main effects?
g) At a 5% significance level, does the speed of agitation have a significant effect on alum yields?
h) At a 5% significance level, does the source material significantly affect the yield of alum?
i) Summarize your conclusions for this ANOVA two-factor factorial experiment.

45. An experiment on the effect diet and exercise have on weight loss involved three different diets and two different exercise regimes. The results of the experiment are summarized below. Complete the ANOVA table and answer the questions that follow.

Factor Information

Factor	Levels	Values
Diet	3	A, B, C
Exercise	2	1, 2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Diet	?	?	?	?	0.0004
Exercise	?	5.0052	5.0052	?	0.0323
Diet*Exercise	?	0.0729	0.0365	0.06	0.9460
Error	?	?	0.6510		
Total	11	56.0573			

a) Complete the missing parts of the ANOVA table above.
b) Identify the factors and levels for this experiment.
c) Give an example of a treatment for this experiment. How many different treatments are there?
d) How many replications were used for this experiment? Why is it necessary to have more than one?
e) What does the provided interaction plot indicate?
f) What is the p-value for the F test statistic related to the interaction effect? What should we conclude about the interaction between these factors?
g) Based on the results of the test for an interaction effect, is it appropriate to test for main effects?
h) At a 5% significance level, does the choice of diet have a significant effect on the amount of weight lost?
i) At a 5% significance level, does the choice of exercise significantly affect weight loss?
j) Use the provided interaction plot to recommend the most effective weight loss strategy/treatment.

46. Convert the ANOVA table for a 2X2 factorial experiment into an ANOVA table with only a treatment and error partition of the sum of squares.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
A	1	6.12500	6.12500	15.68	0.0167
B	1	0.03125	0.03125	0.08	0.7913
A*B	1	0.12500	0.12500	0.32	0.6018
Error	4	1.56250	0.39063		
Total	7	7.84375			

Source	Df	SS	MS	F
Treatments				
Error				
Total				

47. Convert the ANOVA table for a 2X3 factorial experiment into an ANOVA table with only a treatment and error partition of the sum of squares.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
A	1	10.0833	10.0833	23.61	0.0028
B	2	5.5417	2.7708	6.49	0.0316
A*B	2	0.1667	0.0833	0.20	0.8278
Error	6	2.5625	0.4271		
Total	11	18.3542			

Source	Df	SS	MS	F
Treatments				
Error				
Total				

10.4 Answers

38. Yes, the graph indicates there is an interaction effect, since the lines in the graph are not parallel to each another.
39. No, the graph does not indicate that there is an interaction effect, since the lines of the graph all seem to be parallel to each other.
40. Yes, there seems to be a main effect for both fertilizer and sun exposure. The lines for fertilizers indicate that fertilizer A produces a greater yield than fertilizers B and C at either sun exposure level. We cannot tell if the effect is significant, but it appears to be significant. The same is true for the sun exposures. Sun exposure level 1 seems to produce greater yields across all fertilizers. It is possible the difference between sun exposure levels is not significant, but it appears to be significant in the graph.
41. Since the lines in the graph all seem to be parallel, it does not appear that there is an interaction effect.
42. It appears that there are significant differences between sun exposure levels, but the lines for the water schedule levels appear to have little separation or difference. This leads us to suspect that sun exposures levels produce significantly different yields, but different water schedule levels do not produce significantly different yields.

43. a.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Fertilizer	2	12.5903	6.2952	11.330	0.0017
Water	1	0.2222	0.2222	0.40	0.5390
Fertilizer*Water	2	24.5897	12.2949	22.129	<0.0001
Error	12	6.6672	0.5556		
Total	17	44.0694			

b. This information was given:

Factor	Levels	Values
Fertilizer	3	A, B, C
Water	2	1, 2

c. The 3 X 2 is a reference to the fact that the first factor affecting the response has three different levels and the second factor has two levels.
d. Fertilizer A paired with water scheme 1 is an example of a treatment for this experiment. There are six different possible pairings of the fertilizers to water schemes, so there are six different possible treatments.
e. Each treatment was applied three times because we have six treatments and n = 18. It is necessary to have more than one replication per treatment so that we have an error term for comparison.
f. The p-value for the interaction effect is less than 0.0001. It appears that there is a significant interaction effect.
g. Since there is an interaction effect, we will not test for main effects.
h. Since there is an interaction effect, we should proceed to a multiple comparison procedure for all pairs of the treatment means.

44. a.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Material	2	46.0278	23.0139	315.62	<0.0001
Speed	1	0.2222	0.2222	3.05	0.1064
Material*Speed	2	0.3611	0.1806	2.48	0.1258
Error	12	0.8750	0.0729		
Total	17	47.4861			

b. This information was given:

Factor	Levels	Values
Material	3	A, B, C
Speed	2	1, 2

c. Material A agitated at speed 1 is an example of a treatment for this experiment. There are six different possible pairings of the materials to agitation speeds, so there are six different possible treatments.
d. Each treatment was applied three times because we have six treatments and n = 18. It is necessary to have more than one replication per treatment so that we have an error term for comparison.
e. The p-value for the interaction effect is 0.1258. It does not appear that there is a significant interaction effect.
f. Since there does not appear to be an interaction effect, we should test for main effects.
g. Since the p-value is greater than 0.05, it appears that the speed of agitation does not have a significant effect on alum yields.
h. Since the p-value is much less than 0.05, it appears that the source material does significantly affect the yield of alum.
i. The results of the experiment indicate that the source material has a significant impact on alum yields, but the speed of agitation during production does not have a significant effect. It also appears that there is not an interaction effect between these two factors, so the finding that the source material matters will hold regardless of the speed of agitation employed.

45. a.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Diet	2	47.0729	23.5365	36.15	0.0004
Exercise	1	5.0052	5.0052	7.69	0.0323
Diet*Exercise	2	0.0729	0.0365	0.06	0.9460
Error	6	3.9063	0.6510		
Total	11	56.0573			

b. This information was given:

Factor	Levels	Values
Diet	3	A, B, C
Exercise	2	1, 2

c. Diet A paired with exercise routine 1 is an example of a treatment for this experiment. There are six different possible pairings of the diets to exercise programs, so there are six different possible treatments.
d. Each treatment was applied two times because we have six treatments and n = 12. It is necessary to have more than one replication per treatment so that we have an error term for comparison.
e. Since all of the lines are parallel in the plot, it appears there is no interaction effect.
f. The p-value for the F test statistic related to the interaction effect is very large, so it appears that there is no interaction between these two factors.
g. We should test for main effects, since there does not appear to be an interaction effect.

h. Since the p-value is much less than 0.05, the choice of diet seems to have a significant effect on the amount of weight lost.
i. The p-value is less than 0.05, so the choice of exercise seems to significantly affect weight loss at the 5% level of significance.
j. Based on what can be seen in the provided interaction plot, it appears the most affective weight loss strategy is diet B paired with exercise 2.

46. solution

Source	Df	SS	MS	F
Treatments	3	6.28125	2.09375	5.36
Error	4	1.56250	0.390625	
Total	7	7.84375		

47. solution:

Source	Df	SS	MS	F
Treatments	5	15.7917	3.15834	7.3951
Error	6	2.5625	0.4271	
Total	11	18.3542		

Chapter 10 Mixed Review

48. During a CRD ANOVA procedure for an experiment with an unbalanced design comparing five different means, the conclusion is to reject the null hypothesis. If a multiple comparison procedure is to be used to make pairwise comparisons, which procedure would be best Tukey, Bonferroni, or Scheffe? How many comparisons would be made during this procedure?

49. The following data are from an experiment to determine the effectiveness of creatine as a supplement for endurance athletes. Assuming no effect from the interaction between subject and brand, complete the table below in order to test the claim that the brands of creatine all have the same effect on the time to failure. Use a 0.01 significance level.

	Brand A	Brand B	Brand C	Brand D	Totals
Subject 1	120.3	120.5	119.8	116.7	477.3
Subject 2	131.9	132.5	129.6	125.1	519.1
Subject 3	115.2	118.1	113.5	112.4	459.2
Totals	367.4	371.1	362.9	354.2	1455.6

Source	d.f.	SS	MS	F
Brand		53.06		
Subject		471.905		
Error				XXX
Total	11	533.68	XXX	XXX

50. Three types of loans produce the following data:

Loan A	Loan B	Loan C
102	115	125
105	119	115
110	107	110
112	110	105
107	109	117
108	108	120
109	112	121
753	780	813

$$\sum y = 2346, \sum y^2 = 262{,}796$$

Use the treatment totals and the given values to find the **test stat and critical value** to test (at the 2.5% significance level) the claim that the three different loan types produce the same average profit.

51. The following ANOVA table summarizes the analysis of a 3 X 3 experiment that considered the effects of the number of days of study (1, 2, or 3) and the number of exercises completed (10, 20, 30) per study day on Calculus final exam scores. What conclusions can you draw from the analysis?

ANOVA table

Source	*SS*	*df*	*MS*	*F*	*p-value*
Factor 1	991.19	2	495.593	13.94	.0002
Factor 2	6,127.63	2	3,063.815	86.17	6.05E-10
Interaction	198.37	4	49.593	1.39	.2755
Error	640.00	18	35.556		
Total	7,957.19	26			

52. The following computer output was provided for a CRD experiment to determine if the wait times for the lines of four different cashiers are the same on average. What can you conclude from the output?

Source	d.f.	SS	MS	F	P
Cashier	3	102.30	34.1	12.724	0.00000512
Error	41	110.06	2.68	XXX	XXX
Total	44	212.36	XXX	XXX	XXX

53. I ran an ANOVA CRD, F-test to test the claim that the four different paints all produce the same average drying time. We were able to reject that claim, so we then made pairwise comparisons. Use the pairwise comparisons below to put the means in order from lowest to highest, be sure to draw a line above the means that are not significantly different.

$$\mu_A - \mu_B = (-2, 5)$$
$$\mu_A - \mu_C = (7, 14)$$
$$\mu_A - \mu_D = (-10, -3)$$
$$\mu_B - \mu_C = (1, 8)$$
$$\mu_B - \mu_D = (-12, -5)$$
$$\mu_C - \mu_D = (-17, -10)$$

54. Complete the ANOVA table:

ANOVA table

Source	*SS*	*df*	*MS*	*F*	*p-value*
Factor 1		2	802.778	41.05	1.97E-07
Factor 2	3,002.00	2		76.76	1.54E-09
Interaction			23.111		.3521
Error					
Total	5,052.00	26			

55. Considering the provided ANOVA table above, is there a significant effect from the interaction? Should you test to see if there is a significant effect due to either factor 1 or 2? If the answer is yes, is there a significant effect due to either factor 1 or 2?

Chapter 10 Mixed Review Answers:

48. Bonferroni will be used to make 10 comparisons.

49. The test stat is F = 12.1769; the critical value is 9.780. Reject the claim that the brands all have the same effect... The time to failure appears to be different depending on the brand of creatine used.

Source	d.f.	SS	MS	F
Brand	3	53.06	17.687	12.1769
Subject	2	471.905	235.9525	162.4458
Error	6	8.715	1.4525	XXX
Total	11	533.68	XXX	XXX

50. The SST = 258 (with d.f. = 2). The SSE = 456.286 (with d.f. = 18). The test statistic is F = 5.0889, and the critical value is 4.5597. You should reject the claim that the loans have the same average profit.

51. The data indicates that there is no interaction effect, because the p-value is large for the interaction test statistic. However, it seems that both of the main effects are significant because both p-values are small. Therefore, both the number of days of study and the number of exercises completed each day are significant.

52. The p-value is less than any reasonable significance level, so we reject the null hypothesis. The cashiers seem to have different average wait times.

53. $C\ \overline{BA}\ \mathrm{D}$

54. Table:

ANOVA table

Source	*SS*	*df*	*MS*	*F*	*p-value*
Factor 1	1,605.56	2	802.778	41.05	1.97E-07
Factor 2	3,002.00	2	1,501.000	76.76	1.54E-09
Interaction	92.44	4	23.111	1.18	.3521
Error	352.00	18	19.556		
Total	5,052.00	26			

55. Since the interaction test statistic has a large p-value, there is no interaction effect, and we should test for the main effects. Both main effects (factor 1 and factor 2) appear to be significant since both of the p-values are extremely small.