Normal Random Variables

5.4 Normal as Approximation to Binomial

1. Use the normal approximation to calculate the probability that a binomial random variable with n = 32 and p = 0.41 is greater than or equal to 20.

2. Use the normal approximation to calculate the probability that a binomial random variable with n = 51 and p = 0.39 is less than or equal to 25.

3. Gym membership nationwide in 2010 reached an all-time high of 50.2 million members, according to the International Health, Racquet & Sportsclub Association, which surveyed nearly 40,000 individuals on their gym habits. This is approximately 23.9% of the American population between the ages of 15 and 64. If we randomly select 60 people (in the 15 – 64 age range) for a survey, use the normal approximation to estimate the probability that more than 15 of them will have a gym membership?

Answers:

1. Using the formulas for the mean and standard deviation of a binomial random variable we get:
$\mu = np = 32*0.41 = 13.12$, $\sigma = \sqrt{npq} = \sqrt{32*0.41*0.59} \approx 2.78223$. Then we use continuity correction by subtracting 0.5 from 20 since we want to include twenty in our probability, and we get:
$P(X \geq 20) \approx P(X \geq 19.5) = P(Z \geq 2.29) = 0.0110$
2. Using the formulas for the mean and standard deviation of a binomial random variable we get:
$\mu = np = 51*0.39 = 19.89$, $\sigma = \sqrt{npq} = \sqrt{51*0.39*0.61} \approx 3.48323$. Then we use continuity correction by adding 0.5 to 25 since we want to include twenty-five in our probability, and we get:
$P(X \leq 25) \approx P(X \leq 25.5) = P(Z \leq 1.61) = 0.9463$

3. Using the formulas for the mean and standard deviation of a binomial random variable we get:
$\mu = np = 60 * 0.239 = 14.34$, $\sigma = \sqrt{npq} = \sqrt{60 * 0.239 * 0.761} \approx 3.30344$. Then we use continuity correction by adding 0.5 to 15 since we do not want to include fifteen in our probability, and we get:
$P(X > 15) \approx P(X > 15.5) = P(Z > 0.35) = 0.3632$